An Account OF THE ROTULA ARITHMETICA

Invented by

Mr. *George Brown*,
Minister of *Kilmaures*.

Together with Instructions how to use it.

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THE *Lords of His Majestie's Privy Council do hereby Grant to Mr.* George Brown *Minister, and His Heirs and Assigneys, the sole Privilege of Framing, Making, and Selling His Instrument, called* Rotula Arithmetica, *for the space of* 14. *Years yet to come, after the Day and Date hereof. And Discharges any other Persons to make or sell the said Instrument, durcing the space foresaid, without express Liberty and Licence from the said Mr.* Geo. Brown *and his foresaids, under the Paine of* 500. Merks, *besides Confiscation of the* Rotula's *made or sold.*

Extracted by Me,
Sic subscribitur,
Gilb. Eliot, *Cls. Sti. Cons.*

TO THE READER.

Courteous Reader!

WHen first I applied my Mind to publish somewhat concerning my *Rotula Arithmetica*; I designed only (without Preface or Apology) to set down, in the plainest and most homely Dress, such Rules as might render those, who should happen to have both a Book and a *Rotula*, capable by the Help of the one, to make Use of the other; not doubting, but that such an usefull, *Machine* as it is, would be very acceptable to all sorts of persons; men that want Arthemetick being by this means, in the space of four Hours made capable to Add, Substract, Multiply and Divide without any other previous Knowledge, than that of Reading Figures, tho' otherwise such Persons were not able readily to Condescend whether 7 and 4 were 11 or 12: and the ablest Masters, being by the help of this *Machine* rendered more able to performe the most tedious, and most numerous Operations of Addition, Multiplication and Division, with the greatest Certainty, and without all that Rack of Intention, to which they are, by all the other methods hitherto known, obliged. But before I was readie to appear in Publick, I understood by severall Documents, that it is as unfit for new Productions, to go abroad, without the Helmet or cover of a preface, as it's unsafe for a *Highland-man*, to travel amongst those Nighbours, with whom he is at Variance, without the Protection of his Broad *Sword* and *Target*.

Indeed some Persons have been so unjust as to spread a report that the *Rotula* is no new thing, but an old Invention of one *Delamain*, an *English* man, who obtained from King *Charles* I. a Priviledge for his Mathematicall Ring *Anno* 1630. Now this Report, as false as it is, was at first no small prejudice against the *Rotula*, in the Opinion of those, who knew no better, and who had a great deference for rhe sentiments of those, who were the Raisers and Spreaders of this report: and all I my self could say at first, (having never seen *Delamain's* Book nor Instrument) was, that if the *Rotula* had ever been known in the World before, it had never been out of fashion, but had very soon after publication become very near as common as a Balance or an elln-wand, every one, who hath any thing considerable to measure or weigh, having likewise some Accompt to cast.

But tho' this seemed to me a sufficient Demonstration concerning the Novelty made capable to Add, Substract, Multiply and Divide without any other previous Knowledge, than that of Reading Figures, tho' otherwise such Persons were not able readily to Condescend whether 7 and 4 were 11 or 12: and the ablest Masters, being by the help of this *Machine* rendered more able to performe the most tedious, and most numerous Operations of Addition, Multiplication and Division, with the greatest Certainty, and without all that Rack of Intention, to which they are, by all the other methods hitherto known, obliged. But before I was readie to appear in Publick, I understood by severall Documents, that it is as unfit for new Productions, to go abroad, without the Helmet or cover of a preface, as it's unsafe for a *Highland-man*, to travel amongst those Nighbours, with whom he is at Variance, without the Protection of his Broad *Sword* and *Target*.

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But tho' this seemed to me a sufficient Demonstration concerning the Novelty ty of my Invention, yet I was obliged for the Satisfaction of others to procure some Copies, of *De la main's* Book and Projection, that such as were under a mistake might be convinced of the falshood of the foresaid Report, by Comparing both Projections together.

I should be loath to Charge the usage, I mett with in this affair, upon the Score of malice; that base Vice being as far below men of their Generous and Liberal Education, as it lyes out of the Road of my Actions to give them any other Provocation, than that of being an Inventor: Nor can it be Imputed to Ignorance; for they are unquestionably men of Profound Learning, and Knowledge; It must then be chargeable meerly upon the Score of Rashness, for had they but been at the Pains to Compare, both Instruments together, they might have easily perceived the Difference to be as great as that betwixt a Line of *Logarithme* Numbers, and a Scale of equal parts: and

and for this cause I think I may Justly Charge them with Injustice, according to that known Maxime.

Qui statuerit aliquid, parte inaudita altera, licet æquum statuerit, haud tamen æquus fuit.

How reasonably then may the man be charged with Injustice, who passeth not only a harsh, but an unjust Verdict and Censure on a thing he hath not been at the pains duely to examin and consider.

But to proceed; Our worthy Country-man the Lord *Naper* Baron of *Merchistoun*, Invented the *Logarithm*-Tables; and Mr. *Gunter*, an *English* man, Converted these Tables into a straight Lined Scale; after him Mr. *Delamain*, who was *Gunter*'s Scholar converted *Gunter's* Scale into a double Circle, merely to ease men of Compasses: But then his Ring like *Gunter's* Scale, at that time, did only consist of one Line of Numbers, *Sines* and *Tangents*: and as he never dreamed of of performing Addition, or Substraction by the help of his *Machine*, so he Ingenously acknowledges that it was not capable to performe even Division and Multiplication to an Arithmeticall exactness; But many times a man might come short of very near an Unite; Nay He might have added that in Numbers of many Places a man may be some times to seek for Units, and Tens, if not for Hundreds. Nay to render it more certain the anthor requires That, which he proposes as a portable Instrument, to be made of severall Foots or Ellns Diameter, which would render it unweeldy, aud consequently less usefull, either by Sea or Land: and it is not Improbable, that for these defects, that Instrument hath been antiquated, and hath given place to the Double Scale of Proportion, now so much in use.

Now the *Rotula* performes all the four Arithmeticall Operations Arithmetically, and to an Arithmeticall ex-

Exactness, not only of the Integers, but even of the Decimalls, whither *finite* or *Infinite*. So that a man, who can but work by the *Rotula*, may within a little Time and Practice, learn to work by the Pen; if he should chance to want his *Machine*; Nay I believe, when the *Rotula's* are once become common, the mother may teach her Children at home, as much Arithemetick, as may serve them all their lives.

I should now close this tedious business of a Preface, but that I am obliged to give some account of Mr. *Glover*, and his Invention called his *Roue Ariehmetique*,

This Mr. *Glover* is a *Scotish* Gentleman, whose Elder Brother *Thomas*, who was this *John's* Master, was my Scholar about the Year 74. at which time he learned from me that Skill in Numbers and other things which he afterwards taught this Gentleman, and by which both of them have since become famous abroad.

 Now

Now tho' this *Invention* of *John Glover's* be Posterior to my *Rotula*, as appears by the Date of his Privilege, Granted by his Majestie of *France*, which is of the 13. of *March* 1699. whereas my Privilege is granted in *Scotland* on the first of *December* 1698. Yet his comes so far short of mine, that *I* Verily believe, had he seen or gotten a perfect Account of mine before he proposed his own, he would have spared the pains of Publication.

I must Confess that for any thing I yet know, his Tables or Circles for Multiplication and Division (which indeed are very Ingenious, and haue cost him much Thought) are his own; as also his Tables for the Reduction of Pence to shillings, & shillings to Pounds. But in that Part which is common with his *Roue* and my *Rotula*, he seems to haue got some hint of mine; and this I am the more apt to belive because about the time that *I* was bussie in contriving the *Rotula*, there was a very smart Gentleman

tleman, a near friend of his, Scholar with me at *Stirline*.

But that which gives me greater evidence in this particular, is some expressions in his own Book which makes me fancy that he hath, at least, got some imperfect Description of mine, before he contrived that part of his which serves for Addition and Substraction.

For, whereas there is on my fixt Plate 3. Circles, he speaks of three, and yet Immediatly he takes away two of his, & turns them into Tables for Reduceing of Pence into shillings, and shillings into pounds, & these not exceeding the limits of 120. and instead of the third on the *fixt* he gives us nothing but a little segment, about a fifth part divided into parts begining at 0. and ending at 24. which he calls his fixt *Index*: as also whereas my Circle is divided into 100. parts; he Chuses, (to make his differ from mine) 120. as being a common Product of 10. 12. and 20. These Numbers (as he alleadges in the be-

gin-

gining of his 1st. Chapter) being prefferable to all other Numbers whatsoever; and yet near the close of the same Chapter he acknowledges that it would be better to divide the Circle for Addition and Substraction into 100 parts or some Power of 10. and so the Instrument would become universall: All which give me suspicion that in this part, he hath goten, at least some lame account of mine.

Moreover his Instrument is Defective and comes far short of mine, even in Addition; For in his, the Practitioner is obliged to mind or mark down how many Revolutions his moveable Plate makes, and every one being 120, he hath 120 to Multiply by the Number of Revolutions, which is not only troublesom, but likeways dangerous especially in reall Bussiness, where a man whose mind is bussied both about the figures of his Columne, and the points of his moveable plate, is obliged at the same time to mind the severall Revo-

Revolutions of his moveable Plate, of which for every one he forgets, or overlooks, he loses 120 for his Pains; whereas in mine a man is not tied to any such intention above once for 1000. (which is more than any Columne does ordinarly contain) the moveable Plate, at every Revolution both marking and giving notice of the number of Revolutions.

But, besides this, in my *Rotula* the same Circles that serve for Addition and Substraction serve likeways for Multiplication and Division, but in his *Roue* he hath one for Addition and Substraction and ten or eleven for Multiplication and Division and yet tho' the Circles were twice as large, and tho' they contained near twice as many figures as they do, they would be no more than what is necessary to do, what I am able to perform by mine.

Lastly whereas his Tables are confined only to shillings and pence, and these of limited Number not exceeding,

120. There are on the waste, on the middle of my moveable Plate, Tables for the Reduction of shillings, Pence, Farthings, Weights and Measures, be the Numbers never so large; Besides the Decimall Tables for Money, weights Measures and the most ordinary common Fractions: By the help of which six last sort of Tables, the Multiplication and Division of Complex Numbers does become just as easie as that of Integers; without all that tediousness which Mr *Glover* proposes in his Book.

To conclude, what I have said here is no more than was necessary for the Vindication of my own Invention, and to satisfie those, who already are, or hereafter may be misinformed either by the Story of *Delamain*, or Mr. *Glover's Roue Arithmetique*, who for what is Peculliarly his, deserves a good degree of commendation and Encouragement.

CHAP.

CHAP. I.

Concerning the Rotula, *and the Rectification thereof.*

ALbeit in Books of this Nature, it be usuall to prefix a Scheme of the Machine of which they treat; Yet I have thought fit in this, to omit that; because such as have a *Rotula*, need not a Scheme, and such as want one, have no use for a Book; I shall therefore (as briefly as I can) describe the *Rotula*, and then shew You how to use it.

The *Rotula* Consists of two Principall Parts, to wit, a *Circular* Plain moving upon a *Center-pin*, this we call the *Moveable Plate*; and a *Ring*, whose Circles are described from the same Center, this we call the *Fixed Plate*; Because it is fixed to the Box, to secure it from moving about the Center, as the other does.

The

The *Fixed Plate* is divided into three parts by two *Circles*; the Innermost of which is doubled, with a little Interstice for *Peg-Holes*.

Near the Circumference of the Moveable, there is another *Double Circle*, with a small Interstice also betwixt them, for *Peg Holes*.

The space without the double Circle, on the *Moveable*, and within that on the *Fixt*, are both of them equally divided into 100. Parts: and both are Numbered, beginning at 0. 1. 2. 3. and so proceeding in a Naturall Order to 99. all the Divisions being drawen streight from the Center.

On the *Fixt* many of these Divisions are protracted, some only to the middle part; and others run over both: for confining the severall single Coefficents of the Respective Tabular Numbers, to which they are Prefixed, with this Caution when the Coefficients are the same, they are set down in the uttermost part; and when any Number

ber admits of two pair of *Coefficients*, the one Pair is set in the *Midmost* and the other in the *Out-most* Part. Thus against 18. on the *Fixt*, You will find in the *midmost* Part, 2 × 9 (that is two times Nine or Nine times two; This × cross signifieing the word *times*) and 3 × 6 in the outmost. Also on the *Moveable* there is a Segment of a *Circle*, within the *Peg-hole Circle*, beginning at 9. of the Naturall Numbers and ending at 72. This Segment is likewise Divided by the same lines that Divide the outmost *Circle* of Naturall Numbers into equall parts.

On the *Fixt Plate* at the Division betwixt 99 and 0 there is a little bit of Metal Screwed or Rivited, reaching likeways a little further than the *Peg-hole Circle*, on the *Moveable*, this piece of Metall we call the *Stop*: and must always be placed next Your left hand, with the Number 25, or 30. toward your Breast. There is also on the *Fixt*, over against the Numbers

89. or 90. a little *Circle* Divided into 10 equal Parts; with a little *Palme*, which shifts one part at every revolution of the *Moveable*; So that the figure, at which this *Palme* Points, Signifies the Number of Revolutions or Hundreds You have in your Accompt, or the Columne last added

When the Figures of the *Moveable Plate* move towards that side of the *Stop*, which is next the Cypher on the *Fixt*, we call the motion forward; but when they retire from it, the motion is backward.

In Rectification, be sure not to touch the little *Palme*, till the Unites or Numbers beginning with a Cypher on the *Moveable*, be some of them against the Nynties on the *Fixt*; and then turne the little *Palme* to the Cypher of it's Proper *Circle*; after which turn back the *Moveable*, till it will Move no further, and so when the Cyphers of the *Moveable* are just at the *Stop*, as well as the *Palme* of the little *Circle* at it's Cypher; The *Rotula* is *Rectified* and ready for Operation.

I have filled up the Vacant Space in the middle of the *Moveable* with *Decimal Tables*, and Tables *of Common Divisors*, very usefull for those, that have much Bussines, or are in hast.

Onely observe carefuly that the Figures, on the right side of the *Moveable*, are *Top-se turvie*; So that You must alwayes take that which appears to be a *9th.* for a *6th.* and on the Contrary the *6th.* for a *9th.* and mind well, that the Unites are alwaysnext the Center, and the Tens next the Circumference: thus 61. will appear 19. and 91. will look like 16.

CHAP. II.

Concerning Addition.

AFter You have Rectified the *Rotula*, You may easily Perceive that all the Numbers on the *Moveable*, are

lib.	ſs.	d.
345	17	10
976	13	8
158	18	9
746	16	11
843	19	6
977	15	7
865	14	10
743	18	11
896	15	8
739	16	7
478	13	4
287	14	8
958	19	11
549	15	5
378	17	9
486	14	8
297	18	10
684	14	8
1956	18	9
2768	11	6
9875	19	10

are the ſame, with thoſe derectly againſt them, on the *Fixt*; but if You turne any other figure of the *Moveable* to the *Stop*, then will all the Numbers on the *Moveable* be juſt ſo many more than theſe directly againſt them on the *fixt*; for inſtance, if you were to Add any Number whatſoever to 7. firſt, bring that point of the *Moveable* (which is not only directly againſt 7 on the *Fixt*, but indeed the ſame with it) to the *Stop*, and then you'll find all the Numbers on the *Moveable*, 7 more than the reſpective Numbers on the *Fixt*, ſo that

a

againſt 1. on the *Fixt* you have 8. on the *Moveable* which is the Sum of 1. and 7. and againſt 9. on the *Fixt* you have 16. on the *Moveable*, which is Juſt the Summ of 9. and 7. and on this *Theory* depends the Certainty of all your operations: wherefore you muſt take care in bringing up any number of Figures to the *Stop* one after another, to look for all the Items on the *Fixt Plate*, but not on the *Moveable*; Otherwiſe you will miſcarry in your Operation: Example, were you to add the Accompt on the preceeding Page, you muſt begin on the Top of the Columne of Pence, covering the fifth Number with a bodkie Peg, or any other little thing, & then with your *Peg* bring up the Firſt Four Numbers, To wit: 10 8. 9. 11. thus, bring firſt up the point againſt 10. on the *Fixt*, which is ten on the *Moveable*) to the *Stop*, and then that Point on the *Moveable* which is againſt 8 on the *Fixt*, and then that Point of the *Moveable*,

which

which is against 9. on the *Fixt*, and, Lastly that point of the *Moveable*, which is against 11, on the *Fixt*, and you will find the Summ of these four Numbers on the *Moveable*, at the *Stop*, to be 38. but you need not regaird the Summ, till you have done with the whole Column, wherefore shifting your Cover and Proceeding with the next four Figures as you did with the First four Numbers, and so forward till you come to the foot of the Columne you will Find the Summ of your Pence to be just 175. of which 1 is found at the *Palme*, and 75. at the *Stop* on the *Moveable*; these Reduced to shillings (as you are taught in the Chapter concerning Division) do yeild 14 shillings and *7d.* set down your 7 under the Columne of Pennies; and having rectified the little *Palme*, (which must never be Forgot, before you begin a new Columne) set 14. of the *Moveable* to the *Stop*, for the 14. shillings you have to carry.

Then proceed, beginning at the Top of the Column for shillings, bringing up 17, 13, 18, 16, and so forward by four and four, as You did with the Pennies, and Your shillings will amount to 347 *shill.*

These reduced to Pounds, conform to the Directions contain'd in the Chapter of *Division*, do yeeld 17 *lib.* 7 *sh.* Set down your 7 *sh.* under the column of shillings, and having rectifed the *Palm*, put 17 of the *Moveable* to the *Stop*, for the 17 *lib.* you have to carry.

Thence proceed to the Unites of the Pounds, and bring the Figures of that Column up, as you did those of shillings and Pence, and you'll find the Summ of it just 151. the last Figure of which being 1 must be set down under the Integer Unites, and the other two Figures, to wit, 15 must be, after Rectifying the *Palm*, carried as before.

After the same manner You will find the second Column, or *Column of Tens of Pounds* amount to 142, where setting

ting down the last, to wit, the Figure 2, and carrying the other two, to wit, 14. You proceed as before to the Column of Hundreds, the last three Numbers of which having *Thousands* annexed to them, You may bring up alltogether, to wit, 19, 27, 98, and save Your self the Labour of a new Rectification or carrying. This last Sum amounting to 260, must be all set down together in Order, to wit, the 0 under the Hundreds, the 6 under the Thousands, and the 2*d*. Figure before all.

After the same manner, You may Add all other Species whatsoever; providing allways You Divide the lesser Species by their proper Denominators, in Reduceing them to the next greater Species.

In Your first Practice of *Addition*, satisfy Your self with Examples of Integers, where there are no Reductions, till after You have learned *Division*; and then You will find no Difficulty.

If at any Time you Add two Columns

lumns in Integers, you must set down the two Figures at the *Stop*, under the two Columns; and carry that only, at which the *Palm* points. Observe, that how many soever of any one Species are requisite to make one of the next greater: That Number, I call the *Denominator* of the lesser Species.

Thus, in Integers, 10 is allways the Denominator; Because 10 in the Column of Unites, makes but one in the Column of Tens; and 10 in the Column of Tens, makes but one in the Column of Hundereds, *&c.* Nay, the Denominator of Tens is 100. *&c.*

Also, 4 is the Denominator of Farthings, Lippies, Firlots: and Pecks in relation to Firlots; but 16 is the Denominator of Bolls and of Pecks in relation to Bolls; and 12 the Denominator of Pence, and 20 of shillings, *&c.* This is well to be minded, because we may have frequent use to speak of the Denominators of Species.

CHAP. III.

Concerning Substraction.

SUbstraction finds the Difference betwixt two unequal Numbers.

The greater of these two Numbers is called the *Charge*; and the lesser, the *Discharge*.

In *Substraction*, you must allways bring the several Figures of the *Charge*, one after another, together with the respective Figures of the *Discharge*; the one on the *Moveable*, and the other on the *Fixt*, directly against one another; and if the Figure of the *Charge* be equal to, or greater than the respective Figure of the *Discharge*, you have the Remainder at the *Stop*, on the *Moveable*.

But if the Figure of the *Discharge* be greater than that of the *Charge*, then against the Denominator of the Species, on

on the *Moveable*, you have the Remainder on the *Fixt*.

Thus, were I to take 8 from 8, or 7 from 7, having set the one against the other, you have 0 at the *Stop*.

So likeways, if I were to take 5 from 8; having brought 8 on the *Moveable*, against 5 on the *Fixt*: I have at the *Stop*, on the *Moveable*, 3 for a Remainder. But, if I had been to take 8*lib.* from 5*lib.* the Remainder is on the *Fixt*, against 10 (the Denominator of Integers) on the *Moveable*. And 8*d.* from 5 *pennies*, the remainder is on the *Fixt*, against 12 (the Denominator of Pennies) on the *Moveable*. And 8*sh.* from 5*sh.* the Remainder is still on ehe *Fixt*, against 20 (the Denominator of shillings) on the *Moveable*. And 8 *Ounces* from 5 *Ounces*, the Remainder is on the *Fixt*, against 16 (the Denominator of Ounces) on the *Moveable*.

And the Reason of all this is plain; because the *Discharge*, in this case, can-

 not

not be taken off the *Charge*, but off the *Denominator*, which is equivalent to a borrowed one of the next greater Species, and the Overplus, by the very Position of the Instrument, is added to the *Charge*.

Only mind carefully, that as often as the Remainder is found on the *Fixt*, (which allways happens when any figure of the *Discharge* is greater than the respective Figure of the *Charge*,) you must, in that case, esteem the next preceeding Figure of the *Discharge* an Unite more than really it is; Taking 1 for 0, and 2 for 1, and 3 for 2, and so of others.

lib.	*ſs.*	*d.*	
25123478	11	4	*Charge*,
23254906	14	8	*Discharge*.
01868571	16	8	

Thus, in this Example, I bring 8 on the *Moveable* to 4 on the *Fixt*; and because the *pennies* of the *Discharge*, are greater

greater than the *Pennies* of the *Charge*; I look for 12, the Denominator of Pennies on the *Moveable*, and against it, I find 6 on the *Fixt*, for my remainder. These I set down under the Pennies.

Again, because I found my last Remainder on the *Fixt*, I esteem 14 *ſh.* in the *Discharge* to be 15. for which cause I bring 15 on the *Moveable* to 11 on the *Fixt*: and against 20 the Denominator of shillings, I have 16 on the *Fixt*. These I set down under shillings,

Thereafter, for the same Reason, esteeming the 6 *lib.* of my *Discharge* to be 7, I bring 8 (the respective Figure of the *Charge* on the *Moveable*) to it; and (because the Figure of the *Charge*, is greater than that of the *Discharge*) I have 1 on the *Moveable* at the *Stop*, for the Remainder. Thence, because the last Remainder was found on the *Moveable*, I must not change my 0 but bring 7 on the *Moveable* to 0 on the *Fixt*, and the Remainder at the *Stop*, is 7.

And

And proceeding, conform to these Directions, with the rest, I perfect the Operation, finding allways the Remainder, on the *Moveable*, at the *Stop*, when the *Charge* Figure is greater than that of the *Discharge*, or equal to it, but on the *Fixt* against 10. the Denominator of Integers on the *Moveable*, when the *Discharge* Figure is greater than that of the *Charge*.

After the same Manner, and by the same Directions, You may Substract any other Species whatsoever, if You do but carefully mind the Denominators of the severall Species.

CHAP. IV.

Concerning Multiplication.

MUltiplication supposes two Numbers, called *Coefficients*, to find a third, called the *Product*, which Product contains any one of the Coefficients

ents, as oft as the other contains an Unite.

Any one of the Coefficients, especiallie the greater, may be called the *Multiplicand*, the other the *Multiplier*; thus, 3 times 4 is 12. of which 3 and 4 are the Coefficients, and 12 the Product, which Product contains 3, as oft as 4 contains 1.

When one of the Coefficients is 10, 100, 1000. You need no Instrument for Multiplication in Integers, for this is done merely by adding the Cyphers to the right hand of the other Coefficient.

Thus, 10 × 64 is 640, and 100 × 64 is 6400, &c.

But when the Coefficients are all, or many of them, signifying Figures, set the lesser Number under the greater, thus

thus,

```
 7986542   Multiplicand,
    9578   Multiplier.
 --------
 63892336
 55905794
399327 10
71878878
-----------
76495098276   Product.
       9
```

Having Rectified the *Rotula* (with the Pen in your Right Hand ready to Write, and your Left Hand at the *Rotula*, to turn the *Moveable* as shall be necessary) because 8 is the last Figure of the Multiplier, you must look for 8 x every Figure of the Multiplicand, one after another, beginning at the last, but you must not regard the Products on the *Fixt*, but only upon the *Moveable* (nameing allways the Tens of the Product, first, as a single Figure; putting that Figure on the *Moveable* immediatly to the *Stop*, and then the Unites, setting them down on the Paper thus,

First,

First, I look for 8 × 2. and against that, on the *Moveable*, I find 1 and 6. for which cause I set 01 on the *Moveable* to the *Stop*, and 6 I write on the Paper below the Coefficient 8.

Then I look for 8 × 4. and against that, on the *Moveable*, I find 3 and 3. for which cause I turn 03 to the *Stop*, and write down 3 on my Paper.

Then I look for 8 × 5. against which I find 4 and 3. here I put 4 to the *stop*, and 3 again to the Paper.

Thence at 8 × 6. I find 05 for the *Stop*, and 2 for the Paper.

And 8 × 9. I find 07 for the *Stop*, & 7 for the Paper.

And lastly, at 8 × 7. I find 63, all for the Paper, because it is the last Product.

After which, I Rectify again.

Now, as I have gone over all the Figures of the Multiplicand with 8, the last Figure of the Multiplier, so may you do by 7 for the second Product, and 5 for the third, and 9 for the

fourth;

fourth; carefully observing only to set the first Figure, or that next the right hand of every particular Product, under that Figure of the Multiplier, by which it is produced: these particular Products summed up do yeeld the total Product.

I shall subjoyn another Example, and so end with *Multiplication*.

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        94587
       307900
  -------------
     85128300
    662109
   283761
  -------------
  29123337300
```

In this last Example the two *Cyphers* of the *Multiplier* are set to the right of the Unites of the *Multiplicand*, and then multiplying by 9. I set the two Cyphers behind the Product; and so, what was but 9 times before, does now become 900 times the Multiplicand.

You see also the Unites of the Product made by 7, set under 7 of the Multiplier, and the Unites of that made by 3, under 3 of the Multiplier, all the rest duely observing Rank and File.

To conclude this Chapter, and make You prompt in finding your Coefficients: Observe, that all the Products of any Coefficients are contain'd within Ten Times the least of the two, so that all the Products of 2 are within 20, and all of 3 within 30, *&c.*

CHAP. V.

Concerning Division.

SECTION I.

D*Ivision* serves to find a Number shewing how oft the greate[r] of the two given Numbers contains the lesser.

The greater of the given Numbers, we call the *Dividend*: The lesser the *Divisor*; and the Number demanded, or found the *Quotient*,

When as many Figures taken from the left of the *Dividend*; as there are Figures in the *Divisor*, are *æquivalent* to the *Divisor*, or better than it; then we set a *Point* over the last of these, to Determine the first particular Dividend, which for *Brevity*, I shall call the first *Dividual*.

But if as many taken, from the left of the *Dividend*, be less than the *Divisor*, the *Point* must stand over the next *Subsequent* Figure of the *Dividend*, for Determining the first *Dividual*.

Having Determined your *Dividual*, you must refer the first of the *Divisor*, (when they are equall in Number of places) to the first of your *Dividual*; but if they are unequall to the first two of the *Dividual*, and so forward, the second, third and fourth Figures of the *Divisor* to the Subsequent Figures of your

your *Dividual* as they ly in Order; So that in subduction, where you begin at the last of the *Divisor*, you must refer, or *Substract* the Product of it, from the last of the *Dividual*.

The *Remainder* of the first *Dividual* with the next following Figure of the Dividend Yeilds you a 2d. *Dividual*.

So Soon as you have Determined your first *Dividual*, you presently understand, how many Figures you are to have in the *Quotient*; to, wit, one for the *Point*, or first *Dividual*; and one for every subsequent Figure of the Dividend.

Wherefore, if the 2d, 3d, or any other *Dividual* should happen to be less than the *Divisor*; you must put a *Cypher* in the *Quotient* for that *Dividual*; And so (as if it were but a new Remainder) bring down another Figure from the Dividend; to wit, the next following for a new *Dividual*.

I

I ſhall firſt ſhew you how to Divide by one Figure, and then by two, and after that by as many as you pleaſe.

In *Diviſion* by any one Figure, you have nothing to do, but to bring the *Dividual* on the *Moveable* to the firſt *Cell* that occurres, in which the *Diviſor* is a Coefficient; the other Coefficient in the ſame *Cell*, is the *Quotient*, and that (having firſt drawn a Line below the Dividend.) You muſt ſet down under the laſt Figure of your *Dividual*; and the Figure, at the *Stop* on the *Moveable*, you muſt ſet over the ſame laſt Figure of the *Dividual*, for a Remainder. And ſo proceed Rectifying every time before you apply to the next *Dividual*.

Example.

740470
Diviſor, 9) 88654321 *Dividend*.
$9850480\frac{1}{9}$ *Quotient*

Here you ſee, that 8 (the foremoſt Figure

Figure of the *Dividend*,) being leſs than 9, the *Diviſor*; the *Point*, for Determining the firſt *Dividual* ſtands over the ſecond figure of the *Devidend*: So that my firſt *Dividual* is 88: Which being thus Determined, *I underſtand that I am to have in my* Quotient 7 *Figures*; to wit *one for the firſt* Dividual, *and one for every* Subſequent *Figure of the* Dividend.

Theſe things conſidered, and the *Rotula* Rectified, I bring the firſt *Dividual*, 88. on the *Moveable*, to the firſt *Cell*, that occurres on the *Fixt*, in which 9 is a Coefficient; and becauſe the other Coefficient in the ſame *Cell* is 9; *I* ſet that down under 8, the laſt Figure of my *Dividual*; and, having 7 on the *Moveable* at the *Stop*, I ſet 7 over the ſame laſt Figure of my *Dividual* for a Remainder; then I Rectifiie.

Now the firſt Remainder and next *Subſequent* Figure being 76 I bring 76 to the firſt 9 Coefficient, & there I find 8 for my *Quotient*, and 4 at the *Stop* for my my Remainder; these I set down as before, the one under the other, over the last Figure of the 2d. *Dividual*, and then Rectifie.

The 3d. *Dividual* being 45, and having without any *Motion* a *Cell* of 9, directlie against it, I find 5 for my *Quotient*, and 0 for my Remainder.

So that the 4th *Dividual* becomes 04. which being less than 9; I set 0 in my *Quotient* and then

The 4 still Remaining with the next *Subsequent* Figure of my Dividend, making 43; I bring 43 on the *Moveable* to the first 9 Coefficient, and there finding 4 for my *Quotient*, and 7 at the *Stop* for my Remainder.

Having set down these and Rectified, I find my next *Dividual* 72, against a *Cell*. of 9, in which I have 8 for my *Quotient*, and 0 for my Remainder,

So that my last *Dividual* being onlie 01, which is less than my *Divisor* I set nought in my *Quotient*; and 1 the the last Remainder, I set over 9 the *Divisor*

visor at the end of the *Quotient*, with a litle Line betwixt them for a *Fraction*. Thus $\frac{1}{9}$

If any *Divisor* consist only of one signifying Figure & *Cyphers*, you must Divide only by the signifyng Figure, & from the *Quotient* cut off as many Figures towards the right-Hand, as there are *Cyphers* in the *Divisor*, observing, that if the signifying Figure be only an Unite, You have no use for the *Rotula*, or any other Instrument; but meerly to write down the Dividend below the Line in the *Quotient*; & then cut off from it conform to the Number of your *Cyphers*. Example first;

Divisor 1000.) 976583 Dividend.

976|583 *Quotient*.

In this Example, you see the Figures in the *Quotient* are the same with those in the *Dividend*, because the signifying Figure of the *Divisor* is but an Unite. But because there are three *Cyphers* to the Right of the *Divisor*, I have

have cut off three *Figures* from the Right of the *Quotient*, where you see that (as your Dividual *Point* Intimates) You have only three *Integers* in your *Quotient*, namely those to the left-Hand, and the Remainder is a *Decimal Fraction*; or if you will, the *Numerator* of a common Fraction, whose *Denominator* is the *Divisor*; thus:

$$976\frac{583}{1000}$$

```
                        7617
                          .
Example 2d. 800) 478552
                 ------
                 598,19
```

In this Example, I first Divide as if my *Divisor* were only 8, so that I have 5 *Figures* in the *Quotient*, just as if the Dividual *Point* had stood over 7, the Second of the Dividend: But because of the two *Cyphers* in the *Divisor*, I cut off two from the Right of the *Quotient*, and so I understand, that if 800 Men had to receive, or pay out equalie amongst them 478552 *lib.* each Mans

Mans share would come to 598 *lib*. 3 *sh* 9 *d.* and about an half Penny.

Section 2d. Shewing how to Divide by two Signifying Figures

FIRST METHOD.

In this and all *Operations*, where the *Divisor* consists of more signifying *Figures* than one, You must set the *Quotient* to the Right of the Dividend, and the Remainder under the severall Respective *Figures* of every *Dividual*.

Observe in all Subductions, that if the Remainder be either equall to, or greater than the *Dividual* you have taken the *Quotient* too little, or not set down your *Figures* right.

When you are to Divide by any two signifying *Figures*, having first determined your first *Dividual*, & Rectified, look for your *Divisor* on the *Fixt*, & at that, put in a little Peg to mark it.

Then with your other Peg or stile, bring up that *Point* of the *Moveable*

 which

which is against the ſtanding Peg in the *Fixt*, once, twice, thrice, *&c.* till the Number againſt the Peg, with regard to the *Palme*, exceed the *Dividual*, minding only how many times you have brought up that *Point*, and theſe times are the *Quotient* ; & where-ever you find the *Dividual* on the *Moveable*, (which is either at the *Stop*, or betwixt the *Stop* and the *Point*, againſt the Peg in the *Fixt*) there you have the Remainder on the *Fixt*.

As alſo you muſt remember, that when ever the Number on the *Moveable* againſt the Peg in the *Fixt*, is leſs than that on the *Fixt*, that then you are to eſteem your hundreds, one more than that Figure is, at which the *Palme* Points ; becauſe if you ſhould bring up that Number to the *Stop* ; the *Palme* would certainly caſt another hundred: Nay, 00 on the *Moveable*, is always to be reputed 100, when it is not preciſely at the *Stop* ; and ſo you may Judge of every other Number on the *Moveable*,

when it ſtands againſt a greater on the *Fixt*.

Example.

Diviſor, Dividend, *Quotient*.

15) 75986 (5065 $\frac{11}{15}$
098
86
11 *Remainder.*

Having ſet my *Dividual* point, I underſtand that I am to have 4 Figures in my *Quotient*.

I put in a Peg at 15 on the *Fixt* for my *Diviſor* and (the *Moveable* and *Palme* being firſt rectifyed) I bring up conſtantly that *Point of* the *Moveable* which is Directly againſt this *Diviſor* ; ſaying once, twice, thrice, and ſo forward till, at 5 times, I find the Number againſt my *Diviſor* exceed my *Dividual*, and then I put in 5 in my *Quotient* : After which, beginning at the *Stop*, I ſearch betwixt it, & my *Diviſor*-Peg, for my *Dividual* 75, and I find it juſt at the *Stop* : and againſt it on the *Fixt*, finding a *Cypher* for my my Remainder; I understand that 5 times 15 is just 75. and therefore *I* set down 0 under 5, the last Figure of my *Dividual*, and to it I bring down 9 the next subsequent *Figure* of my *Dividend*, for the second *Dividual*.

Now because this 2*d*. *Dividual* is less than the *Divisor*, *I* put 0 in my *Quotient*; and to the 2*d*. *Dividual* as a mere Remainder, *I* bring down the next *Figure* of the *Dividend*, to wit 8, and so I have 98 for a third *Dividual*.

Now you may either Rectify, and begin *de Novo* for 98, as you did for 75: Or, because this *Dividual* is greater than the last, to wit 75, you may proceed: Bringing up the *Divisor* once more, and saying 6 times; so have you 6 for your *Quotient*. And against 98 on the *Moveable*, you have 8 on the *Fixt* for your Remainder.

When you have set down 8 on the Paper under 8, the last *Figure* of the third *Dividual*; you bring down to it,

it 6, the last *Figure* of the *Dividend*, so have you 86, for a new *Dividual*, and having Rectified; you find the *Quotient* 5, and the Remainder 11, after the same manner as you did the first.

If after this manner, you *Divide* the Summ of your *Pennies* in *Addition* by 12, and that of your *shillings*, by 20, you will reduce the first to *shillings*, and the second to *Pounds*, in the *Quotients*; and the remainders are *pennies*, or *shillings*, according to the Nature of the Summs Divided: The same may be said of every other Species, if it be Divided by its Proper Denominator.

SECOND METHOD.

I shall here likeways shew you how to do the same by the Tables of common *Divisors* on the middle of the *Rotula*; which Method will likeways be sometimes very usefull in long *Divisions*, either by two, or more Figures.

Suppose then that the Summ of your *pennies* were 798. You must in reducing

cing them to *shillings*, *Divide* them by 12, because 12 is the *Denominator* of *pennies*, wherefore having set them down as in the Margine, & Marked your first *Dividual*, You must look for your *Dividual* 79 in that Table, whose first Number is 12, & if you can not find 79, you must take that which is next to it, but less; and that you will find to be 72, against which you have 6 for your *Quotient*; as you find in the Columne on the Left of the Tables, under the Letter *N*. That 6 you put in your *Quotient*; and then *Substracting* 72, the Tabular Number, from 79, the *Dividual*, you set down the 7 that remains under the 9 of the *Dividual*. And then bringing down 8, the next of the Dividend, to 7 the Remainder, you have 78 for the 2d. *Dividual*; the next to which in the Table is 72, which gives another 6th. Figure for your *Quotient*, and 6 for your Remainder; So that in

```
12) 798 (66
     78
      6
```

798

798 pennies; you have just 66 *shilling*, and 6 *d*. The last to set down under pennies, and the first to carrie to your *shillings*.

After the same manner you reduce shillings to pounds, by the Table, which begins with 20: and Weights & Measures by the Tables of 16, 4, and 28, according to their several Denominators

You may likeways in long *Divisions*, to prevent many turnings of the *Rotula*, at one Fetch set down the Nine Multiplies of any two Figures, and so Divide as by the Table,

```
Thus.        19)  796857(41939
             38    36
             57    178
             76     75
             95     187
            114      16
            133
            152
            171
```

In This lesson I put 19 to the

Stop

Stop, and then against 19 on the *Fixt*; I have 38 on the *Moveable*, and against 38 on the *Fixt*, I have 57 on the *Moveable*. These I set down, and proceed, seeking 57 on the *Fixt* I find 76 on the *Moveable*, and at 76 on the *Fixt*, I find 95 on the *Moveable*; then I sett down 76, and 95, also *I* find at 95 on the *Fixt*, 14 on the *Moveable*, and at 14 on the *Fixt*, *I* find 33 on the *Moveable*, and at 33 on the *Fixt* I find 52, these likeways *I* set down & then in the Last place at 52. on the *Fixt I* find 71 on the *Moveable*, which having set down, *I* distinguish my nine Numbers into threes by two Lines.

Now because my *6th*. Number is less than my fifth; *I* understand by that, That here I must add one hundred, which hundred continues invariable till the Unites and Tens, of a following Number grow less than those of a preceeding, & then I must add one more to the hundreds,

Having thus made my Table I search

search in it, for the nearest Number to 79; my first *dividual*, and finding 76, The fourth Number I set 4 in my *Quotient*; and substracting the Tabular Number 76, from 79 I set down the Remainder 3 under the 9, of my *dividual*.

To this bringing down 6, I have 36 for my second *dividual*; for which looking in the Table I find 19, the nearest, which being the first number, I set 1, in my *Quotient*; and taking 19 from 36, I set down 17 the Remainder under 36, then bringing down 8 of the dividend to 17; I have 178 for my next *dividual*; the next to which in the Table is 171, which because it is the *9th* Number, I put 9 in my *Quotient*, and the Remainder 7, *I* set under 8, and so proceeding after the same manner with the two subsequent Figures of the dividend; *I* find my *Quotient* to be 41939, & my Remainder to be 16.

After

After the Method here, proposed for making your little Table, You may Examin the exactnes of the Tables of common *Divisors*, and so understand, whether upon Occasion you may trust them or not.

Section 3d. Shewing how to *Divide* by any Number of *Figures* whatsoever.

In this part, having first Determined the first *Dividual* observe whether the *divisor* and *dividual* be equall in Number of places. *For* if they are equall, you must Refer, or Compare, the first two of the *Divisor* to the first two of the *Dividual*: But if the *Dividual* have one *Figure* more than the *Divisor*; you must refer the first two of the *Divisor* to the first three of the *Dividual*.

Then you must (either by the first or 2*d*. Method, of Dividing by two *Figures*) *Find* out how oft the foremost two *Figures* of the *Divisor* is contained in the Respective *Figures* of the *Dividual*,

(whether the foremost two or three;) And having found your *Quotient*, you must set it under the *Divisor* (at convenient distance before the following Product) with the signe of Multiplication (to wit, ×,) after it, and after that the Letter *d*, and after that the signe of equalitie to wit (=) for instance, suppose the *Figure* found for the *Quotient* were 7, you must under the *Divisor*, as you see in the following Example, Set it down Thus, (7 × *d* =) which signifies that 7 times the *Divisor* is just equal to the Number that followes the signe of equalitie: This done, Multiplie the *Divisor* by the *Figure* found, and set the Unites of the Product under the last *Figure* of the *Dividual*, and the rest in Order.

If this Product do not exceed the *dividual*, then you are sure that the *Figure* found is the true *Quotient Figure*, for which cause you must write it in the *Quotient*, and then having Substracted the Product from the *dividual*; you

you must bring down the next *Subsequent* Figure of the Dividend to the Right of the *Remainder*, so have you a Second *Dividual*.

But if the *Product* should happen to be greater than the *Dividual* (which will sometimes fall out) do not Expunge it; (for it may be afterwards usefull) but abate an Unite from the Coefficient allreadie found; so have you the true *Quotient*, by which when you have found it, Multiply the *Divisor*, & Substract this last *Product* from the *Dividual*, & Proceed conform to the Directions allreadie given. This is the far shortest Method of any I know for finding the true Figure for the *Quotient*; & does abundantly compense the little Trouble a Man is at in making sometimes two *Products*, by freeing a Man from all that tedious Chain of thought, by which he is Obliged to compare everie severall Figure of his *Divisor*, with the Respective Figures of his *Dividual*, especiallie, when they consist of many Figures. Onlie when you see the first Remainder, or the

the Difference betwixt your Tabular Number, and the Respective Figures of the *Dividual*) palpablie too little to make the next Subsequent Figure of the *Dividual* equall, or answer as many times the third of the *Divisor*, you may abate an Unite from the *Quotient* Figure, allreadie found before you make your first *Product*.

EXAMPLE.

1598)		7620475	(4768
30	4 × *d.* =	6392	
45	2d. *Divid.*	12284	
60	7 × *d* =	11186	
75	3d *Divid:*	10987	
190	6 × *d* =	9588	
105	4th *Divid*:	13995	
120	9 × *d* =	14382	
135	8 × *d* =	12784	
Last *Remainder*		1211	

In this Example, because the *Divisor* and first *Dividual*, have an equall Num

Number of Places, I refer 15 the first two of the *Divisor*, to 76 the first two of the *Dividual*: And then by the *Rotula*, or little Table made by the help of the *Rotula*, I find 5 × 15, or 75 in 76: But then the Remainder, which is but 1, makes the third Figure of the *dividual* but 12, which being less than 5×9 (the third Figure of the *Divisor*,) I abate an Unite from the first *Quotient*, & so take it only 4 times; then I Multiplie the *Divisor* by 4 and disposing the *Product* duly under the *Dividual*; I Substract the *Product* from the *Dividual*, & to the Right of the Remainder, I bring down the next Subsequent Figure of the Dividend; so have I a second *Dividual*. Now the *Divisor* having but 4 Figures, and the second *Dividual* 5, I refer the first two of the *Divisor* to the first 3 of the *Dividual*, to wit, to 122. Wherefore searching the little Table for 122; I find 120, the nearest to it, which would yield me 8 for my *Quotient*, but the Remainder 2, makes the fourth

fourth Figure of the *Dividual*, only 28 which is much less than 8 × 9 the third Figure of the *Divisor*, wherefore I content my self with 7 for the true *Quotient* and proceed as before.

In the third *Dividual*, I find by the foremost 3 figures 109 that I may have 7 times 15 (the foremost two of the *Divisor*) in that part of the *Dividual*, but looking back on 7 × *d* under the 2d *Dividual* I find, That that Product exceeds the 3d *Dividual*, for which cause I take 6 instead of 7 for my true *Quotient*.

In the 4th *Dividual*, I have shewed you how to do, in case you should chance to take your *Quotient* an Unite bigger than it ought to be: For finding 9×*d* greater than the *Dividual*, I Substitute 8×*d* under it, & Substracting that from the *Dividual*; I have 1211 for my last Remainder.

I have now gone through the 4 common Rules, and if what *I* have said be well understood, a Man may propose as many Examples as he pleaseth, and

perform

perform them with the like ease: And I am confident, that a Man of a very ordinarie Capacity may learn what concernes *Addition* and *Substraction* of all sorts or Species: *Multiplication* in Integers; And *Division* by one or two Figures in the space of one Hour, or at most of one Hour and a half, so that he hath remaining two houres and a half, for accomplishing this last lesson of *Division* by more Figures, which I doubt not but he will be able to do in shorter space, so that conform to what I undertake, the Scholar learnes the 4 Com-Rules in the space of 4 Houres.

But before I leave this, & to shew you how Copiously useful the Instrument is, I will set you a Method of Dividing by many Figures, in which you may (without Table or setting down a Product) come both at your *Quotient* and Remainder, after the same manner as you ought to do, if you were Dividing by the Pen.

To this purpose you may provide your

self at first with a Label of Cart or Paper, writting on the right edge of it, the Figures beginning at 0, 1, 2, and ending at 9, This we call the Labell of prefixes as you have it in the Margine.

PREFIXES
0
1
2
3
4
5
6
7
8
9

Now suppose the following Example were proposed, having first rectified the *Rotula*, & with a Point determined the first *Dividual* proceed after this manner.

```
1986 (15036948) 7571
        11349
         14194
          02928
           0942
```

First *I* bring 15, the foremost two of the first *Dividual* on the *Moveable*, (with a little Peg to stay there till I have found my *Quotient*,) to the first Cell, in which 1 (The foremost Figure of my *Divisor*) is a Coefficient, and there I have in the same Cell, 9 for a *Quotient*, and at the *Stop*, 6 for a Presix; wherefore I lay 6 on the Label of Presixes over 15, and so (0) the third of my *Dividual*, becomes 60: Now because 9 is my *Quotient*, and 9 is also the 2d. Figure of my *Divisor*, I look for 9×9 on the Fixt, and finding it exceeds 60 the 2d: Number of my *Dividual*. I therefore shift the Point with the Peg in it forward, to the next Cell of (1) and there having 8 for my *Quotient*, and 7 for a Presix, I alter my prefix to 7, and then consider, whether this *Quotient* 8 × 9 the 2d. Figure of the *Divisor*, exceeds 70, the respective number of the *Dividual*, & because 8×9 is greater than 70, I shift my standing Peg once more to the next Cell of (1) & there having 7 for a *Quotient*, and 8 at the *Stop* for a Presix; I am confident, (that the Remainde

der being greater than the *Quotient*) 7 will serve for all the following Figures o the *Divisor*, and so I put 7 in my *Quotient*, then laying aside your Label till you come to seek a new Figure for the *Quotient*. You must in the Subducti-on observe to take 7 times every Figure of the *Divisor*; beginning at the last from the respective Figures of the *Dividual*, that is to say the last from the last, and the last save one, from the last save one, and so forward till you take 7×, the foremost of the *Divisor*, from the foremost one or two Figures of the *Dividual*, according as they fall out to be one or two Thus, I first rectify, and take out the Peg, & because I must take 7×6 of the *Divisor* from 6 of the *Divial*, I with my Peg bring up 7×6 on the *Moveable* to the *Stop*, and searching for a Number on the *Moveable*, having 6 Unites; *I* find the first that occurres to be 46 by which I understand, that I am to carry 4 to wit, the tens of 46, & because I have 4 on the *Fixt*, against 46 on the Mo

Moveable, *I* understand, that I must set down 4 as a Remainder under 6 the last *Figure* of my *Dividual*, and so turning back the *Rotula* till 4 on the *Moveable* appear against (0) at the *Stop*; I thereafter write down 4 below the 6 of my *First Dividual*.

Then I bring up to the *Stop*, that *Point* of the *Moveable*, which is Directly against 7×8 of my *Divisor*, and looking as before for 3 Unites I finde 6 tens to, carry, and 3 on the *Fixt* to set down, these carried, and set down as becomes I next bring up 7×9 of my *Divisor*, & at (0) Unites on the *Moveable*, *I* have to carrie 7 tens, and on the *Fixt* (1) to set down under (0) of my *Dividual*, of these the 7 carried, and the (1) set down.

In the last place *I* bring up 7×1 of my *Divisor*, to the *Stop*, and on the *Fixt* *I* have 1 (against 15 the foremost two of the *Dividual*) for my Remainder, which 1, *I* set under 5, the Unites of the [illegible] and so having finished the first Sub

Subduction, I rectify and bring down 9 of the Dividend, to the right of the first Remainder, for a second *Dividual*.

If you comprehend what I have said on the first, you may easily, and after the same manner go through with the other there *Dividuals*, and so perform the whole business your self.

If the *Practitioner* curiously observe the severall Operationes by the *Rotula*, he will discover them to be so Natural, that with a carrefull Practice, he may come to such a Habit as will render him capable to do his business with the Pen when he wants his *Rotula*. Tho' *I* must confess; That the ablest Masters are not capable without it, to do any considerable business with that dispach, Certainty and Exactnes, and with so little Trouble to the Head; as by the help of the *Rotula*, they may perform with the greatest ease.

If any difficulty Occur, in what hath been hitherto delivered, such as have the Opportunity, shall not want what

Moveable, *I* understand, that I must set down 4 as a Remainder under 6 the last Figure of my *Dividual*, and so turning back the *Rotula* till 4 on the *Moveable* appear against (0) at the *Stop*; I thereafter write down 4 below the 6 of my *First Dividual*.

Then I bring up to the *Stop*, that *Point* of the *Moveable*, which is Directly against 7×8 of my *Divisor*, and looking as before for 3 Unites I finde 6 tens to, carry, and 3 on the *Fixt* to set down, these carried, and set down as becomes I next bring up 7×9 of my *Divisor*, & at (0) Unites on the *Moveable*, *I* have to carrie 7 tens, and on the *Fixt* (1) to set down under (0) of my *Dividual*, of these the 7 carried, and the (1) set down.

In the last place *I* bring up 7×1 of my *Divisor*, to the *Stop*, and on the *Fixt* *I* have 1 (against 15 the foremost two of the *Dividual*) for my Remainder, which 1, *I* set under 5, the Unites of the [illegible] and so having finished the first Sub

Subduction, I rectify and bring down 9 of the Dividend, to the right of the first Remainder, for a second *Dividual*.

If you comprehend what I have said on the first, you may easily, and after the same manner go through with the other there *Dividuals*, and so perform the whole business your self.

If the *Practitioner* curiously observe the severall Operationes by the *Rotula*, he will discover them to be so Natural, that with a carrefull Practice, he may come to such a Habit as will render him capable to do his business with the Pen when he wants his *Rotula*. Tho' *I* must confess; That the ablest Masters are not capable without it, to do any considerable business with that dispach, Certainty and Exactnes, and with so little Trouble to the Head; as by the help of the *Rotula*, they may perform with the greatest ease.

If any difficulty Occur, in what hath been hitherto delivered, such as have the Opportunity, shall not want what

what help I can afford them. But because in what followeth, a Previous knowledge in *Decimal* Fractions, is supposed; If such as want that find any difficulty, they must be at the paines, or use a Master, as they find most convenient, for the attainment of that knowledge before they make any further or Considerable Progress.

CHAP. VI.

Concerning COMPLEX NUMBERS.

I Call those *Numbers Complex*, that consist of Intigers and Fractions, or small Denominations, such as *lib.* sh. *d.* or *Stones lib.* of Weight and *Ounces. Chalders, Bolls* and *Pecks*, &c. *Elnes, Half quarters*, or *Eight parts*, &c.

If instead of these Fractions, or Denominations, you annex the *Decimal* to the right of the proper *Integer* distinguishin

guishin

guishing betwixt them with this mark (⌞) called a *Decimal Line*, you may Multiplie or Divide, as if the whole Number represented by all them Figures were an Integer. In adding *Decimals*, You must take care that those Figures next the *Decimal Line* make one Columne, & the rest in order. Thus were I to Joyn the *Decimal* of 7*sh.* which is ⌞35 to the *Decimal* of 9 *d.* which is ⌞0375 *I* must state them Thus

⌞35
⌞0375

And so the *Decimal* of 7*sh.9d.* is ⌞3875

Observe that the *Decimals* for *Pence* and *Farthings*, where the last Figure is 3 or 6 as also those for third parts or sixth parts (whose last Figures are likeways 3 or 6) are all Infinites; Thus the *Decimal* of one *d.* is 00416 the last *Figure* of which may be reiterated in *Infinitum*, or as oft as the Rules of Operation do require; for which *I* refer you to my Compendious, but compleat ſyſtem of *Decimal Arithmetick* : But leaſt that ſhould not come to your hands I ſhall here ſubjoyn a few *Rules*.

Firſt you muſt not limit Infinites ſhort of *Decimal* thirds.

2ly You muſt not limit infinites unleſs they contain one of the Reiterated Figures ; Thus becauſe the *Decimal* of 4 *d.* is 01666 *&c.* I may upon occaſion ſatisfy my ſelf with 016 and 8 *d.* being 03333 I cann't ſatisfy my ſelf with 03 but 03 as alſo the *Decimal* for one *penny* being 0041666 *&c.* I can ſatisfy my ſelf with no leſs than 00416

3ly Infinites in *Addition* and *Subſtraction*, muſt exceed the longeſt finite at leaſt by one Step towards the right Hand, thus, were I to Joyn one *penny*

viz.	0041666
To three farthings viz.	003125
The Summ would be	0072916

In which you may obſerve, that I have Reiterated the laſt Figure of the *Decimal* for one *d.* twice.

4ly

4ly. Infinites muſt in *Addition* & *Subſtraction*, be equall in Number of places; Thus were *I* to ſet down a *Decimal* for $4\frac{1}{2}$ *d.* tho' 016 might ſerve for 4*d.* yet

becauſe the *decimal* of $\frac{1}{2}$ *d.*	is	002083
The *decimal* of 4 *d.*	is	016666
So that the *decimal* of		018750

$4\frac{1}{2}$ *d.* is 001875 throwing away the *Cypher* after the *Decimal*, as a thing of no value.

Laſtly, The Summ, difference and *Product* of infinites is infinite, unleſs they end in a *Cypher*.

Wherefore the Summ, Difference and *Product* of the laſt Columne or Figure of an *Infinite*, muſt be reckoned on the *Segment* within the *Peg holes* of the *Moveable*, and not on the whole *Circle* without ; Thus in the laſt Example you ſee that 6 & 3, which make 9 on the whole *Circle*, make 10 on the *Segment* ; & therefore I ſet, 0, and carrie one : But in all the reſt of the *Figures*, I regard the whole *Circle*, but not the *Segment* ; the Reaſon, of which is manifeſt from the 1ſt. Chapter of my *Decimal* Syſteme.

In Cutting of your *Decimals* after Operation, you must obſerve.

Firſt that the *Decimal* of the Summ, or difference, muſt in Number of places, be equall to the longeſt *Decimal* of the Items.

Secondly, That In *Multiplication* the *Decimal* of the *Product*, muſt be equall, in Number of places, to thoſe of both *Coefficients*.

Thirdly, That in *Diviſion*, the *Decimal* of the Dividend alone, muſt equall thoſe of the *Diviſor* & *Quotient* both together.

Fourthly, All Numbers, either Actually have a *Decimal* annexed to them, or muſt be ſuppoſed to have as many *Decimal Cyphers* as may be neceſſary, nay Finit *Decimals* themſelves, muſt ſometimes be ſuppoſed to have *Cyphers* following them; and on the other Hand Integers, muſt be ſuppoſed to have *Cyphers* to the left: For theſe *Additional Cyphers* do neither increaſe, nor diminiſh the true value of any Number.

Multiplication by ſuch pure Numbers as

10,

10, 100, 1000, &c. Is done meerly by ſhifting the *Decimal* Line of the *Multiplicand*, ſo many *Steps* nearer the Right, as there are *Cyphers* in the *Multiplier*; only if the *Multiplicand* be Infinite you muſt Reiterate the laſt Figure. Thus were it Demanded; How much would pay 100000 Men, to give them 3 Farthings a Piece? Now the *Decimal* of 3 Farthings is ,003125, and becauſe there are five *Cyphers* in the *Multiplier* the Anſwer is 00312,5 So that 312 *lib.* 10 *ſh.* will juſt pay 100000 Men at 3 Far things *per* Piece; here you have no Reiteration, becauſe the *Decimal* is finite.

But at 4 *d.* *per* Piece, the *Decimal* of which is an Infinite, to wit, ,016 You muſt Reiterate the laſt Figure, and then you'll find the Anſwer for ten Men to be ,166 That is 3 *ſh.* 4 *d.* But for 100 01,666 That is 1 *lib.* 13 *ſh.* 4 *d.* and for 1000, 016,666. *id eſt* 16 *lib.* 13 *ſh.* 4 *d.* for 10000, 0166,666 *id eſt* 166 *lib.* 13 *ſh.* 4 *d.* after the ſame manner you may by the *decimal* Tables for *pence* and *ſhillings*, at one look turne any pure Number into

their

their Proper *Integers*; Thus were 7000*d.* to be turned into *Pounds*, take the *Decimal* for 7 *d.* to wit, 02916 & by shifting the *Decimal* Line one *Step*, you have the Value of 70 *d.* to wit, 02916 which is 5 *sh.* and 10 *d.* But if you shift it two *Steps* to the right Hand, you have the Value of 700 *d.* namely 02916 which is 2 *lib* 18 *sh*, 4 *d.* and if you shift it 3 *Steps*, you Reduce 7000 *d.* to their proper *Pounds* and *shill*, namely 029166 that is 29 *lib.* 3 *shill.* 4 *d.*

I have contrived a little Pocket-Book whereby you may with the same ease convert any pure Number of the Species Current in this Kingdom, into *pound Scots* or *Sterling*, at one look.

You may use the Table of *Ounces* to the same purpose, in turning *Drops* to *Ounces*, *Ounces* to *Pounds*, and *Bolls* to *Chalders*; All which requires an exact knowledge of *Decimals*. Example; The *Decimal* for 6, *Ounces* or *Pecks*, is 375 and consequently 60 *Pecks* makes 375

that

that is 3 *Bolls*, 12 *Pecks*, but 600 *Pecks* make 375 that is 37 ½ *Bolls* & 6000 *pecks* are just 375 *Bolls*.

In *Division* by 10, 100, 1000, *&c.* Just contrary to *Multiplication*, you must remove the *Decimal Line* so many *Steps* nearer the left, as there are *Cyphers* in your *Divisor*: Thus were 2916 *lib.* 13*sh.* and 4 *d.* to be Divided amongst 10, 100, 1000, or, 10000 Men. You must first set down the Dividend 2916666 and then the severall shares of 10, 100, 1000, 10000, or, 100000, will appear as followeth.

Divisor	10	29166	is	291*lib.*	13*sh.*	4*d*
	100	29166	is	29	3	4
	1000	2916	is	2	18	4
	10000	2916	is	0	05	10
	100000	02916	is	0	00	7

I must refer such as desire further Satisfaction in this particular to my *Compendious*, but compleat *System* of *Decimals*.

Chap

CHAP. VII.

Concerning the Rule of 3.

IN the *Questions* of this Rule, there are allways three Numbers, either expresly given or supofed, to find a Fourth.

Of the given Numbers, there are allways two of the fame fort, Species or Notion, & thefe we fhall call *Relative Numbers*, & the other or third Number, which falls under a different Notion, we fhall call the fingular Number: and the fourth Number demanded (which is allways of the fame Species, or Notion with the fingular Number) we call the Anfwer; becaufe, when it is found, it Anfwers the Queftion.

Now to fave you the trouble of two Rules, one Direct, and another Reverfe: I fhall lay down fuch eafie *Directions*, as may render a Man capable to Anfwer Queftions of either, without any fuch Diftinction.

In difcerning the *Divifor*, (which is all the difficulty, and which muft allways be one or other of the Relative Numbers) you muft carefully confider, whether the Anfwer ought to be greater, or leffer than the fingular Number For,

1*ft*. When the Anfwer ought to be greater than the fingular Number, then the leaft of the Relative Numbers muft be the D*ivifor*.

2*ly*. But if the Anfwer ought to be lefs than the fingular Number, then the greateft of the Relative Numbers muft be the D*ivifor*.

Having difcovered the D*ivifor*, fet it down next your Left-Hand, and the other two, (which we now call *Coefficients*) at a convenient Diftance; the one, (it matters not which) before, the other after[::]which we call the figne of Proportion.

The Numbers thus difpofed, Multiply the *Coefficients* and Divide the *Product*, by the D*ivifor* and the *Quotient*, yields you the Anfwer; Only obferve, that when the *Divisor* is an Unite, there will be no use for *Division*; and on the other hand, when one of the *Coefficients* is an Unite there, will be no *Multiplication*.

As also, if one of the *Coefficients* have an Infinite annexed to it, be sure to make that the *Multiplicand*.

The following Examples, shall Illustrate what hath been said.

EXAMPLE, 1st.
Concerning the Prices of GOODS.

IF 3 Ells cost 10 *L.* what will 17 $\frac{3}{4}$ Ells cost? In this Example 3 and 17 $\frac{3}{4}$ are the Relative Numbers, because they are both of the same sort, to wit, Ells, and 10 is the singular Number; now because the Price 17 $\frac{3}{4}$ Ells, must be greater than the price of three Ells to it, than 10 *L.* that is the Answer; must be be greater than the singular Number; I thereby understand, that 3 the least of my Relatives must be the *Divisor*,

for which cause I set the Numbers as followeth. Taking instead of $\frac{3}{4}$ it's *Decimal* 175.

3 *Ells* ; 17,75 *Ells* :: 10 *L.*
10
3) 177,50
59,166

So that the Answer is 59 *L.* 3 *sh.* 4 *d.*

You see I first *Multiply* the *Coefficients*, & then Divide the *Product* by 3 the *Divisor*, so have I the Answer: You may convert the Question, and so prove your Work, Thus,

2d. EXAMPLE.

If 17$\frac{3}{4}$ Ells cost 59 *L.* 3 *sh.* 4 *d.* what will 3 Ells cost?

17,75 *Ells*, 3 *Ells* :: 59,166 *L.*
3
17,75) 177,500 (10 *L.* ans.
00000

You see in the *Division*, that I ought to have 3 Figures in my *Quotient*, as the *Point* intimates, of which one ought to be a *Decimal*, but that being a *Cypher* I did not think it necessary to set it down.

K 2

3d. EXAMPLE.

At 3 *sh.* 4 ½ *d. per lib.* what will 345 ½ *lib.* of any thing come to?

The *Decimal* for 3 *sh.* 4 *pence* is ,166666
and that for ½ *d.* ,002083

So that 3 *sh.* 4½ *d.* is ,168750

Then conform to the Rule, the lesson will stand as followeth.

1 *Lib.* ; ,16875 *L.* :: 345,5 *Lib.*

```
        ,16875
         345,5
         84375
        84375
(50)   67500
      50625
      58,303125 ans. 58 li. 6 sh. 0 3/4 d.
```

You see in the lesson, there is no *Division*, because the *Divisor* is an Unite.

4th. EXAMPLE.

If 345 ½ Ells cost 58 L. 6 *sh.* 0 ¾ *d.* what will one Ell cost?

3455 Ells ; 1 Ell :: 58,303125 *L.*

```
3455) 58,303125 (,16875 L.
        23753
         30231    Answer 3 sh. 4 d.
          25912
           17275
(7)         0000
```


In this *Example*, you see that one of the *Coefficients*, being an Unite: There is no *Multiplication*.

5th. EXAMPLE.

If $\frac{3}{4}$ Ells cost $\frac{4}{5}$ *lib.* what will $\frac{5}{6}$ Ells cost?

,75 Ells ; ,8 *L.* :: ,833 Ells

```
                 ,8
(75) ,6866 (,9142
       116
        316
         166
          166
```

Answ. 18 *sh.* 3 ¼ *d.* & some little more.
Because the last *Dividual* is the same the *Quotient* is Infinite.

In this last Example, the *Decimal* of $\frac{5}{6}$ being Infinite; The *Product* of the dash'd

Figure

Figure, is reckoned on the *Segment*, but the *Product* of all the rest, is reckoned on the Integer-Circle,

Item, because the Dividend is Infinite, in *Prolongation* of the *Work I* have Reiterated the last Figure.

6th. E P A M P L E.

At 5 $\frac{1}{2}$ *per Cent*, what will 347 *lib.* 13 *sh.* 4 *d.* pay *per Annum*?

In this lesson, observe, that tho' all the Numbers be of one Denomination or kind of things, yet (the $5\frac{1}{2}$ *L.* falling under a different Notion, to wit, that of Interest; whereas the other two, to wit, the *Cent*, or 100, and 347 L. 13 *sh.* 4 *d.* are Principal Summs) the $5\frac{1}{2}$ is the Singular Number, and so 100 is the *Divisor*.

Principal	*Interest*		*Principal*
100	55	::	347,666
			55
			1738333
			1738333
Answer 19 *lib.* 2 *sh*, 5 *d.*			19,121666 *l:In.*

and a little more than $\frac{1}{2}$ a *Farth.*

In

In this you have 6 *Decimals* in the Answer to wit, four for those of the *Coefficients*, and two for the *Cyphers* of the *Divisor.*

7th. E X A M P L E.

At 4 *sh.* $10\frac{1}{2}$ *per Crown*, how many *Pounds Sterl.* must one have for $758\frac{1}{2}$ *Crowns.*

1 *Cr.*	5	7585	::	,24375 *lib. St.*
				758,5
				121875
				195000
				121875
				170625
Answer. 184 *lib.* 17 *sh.* $8\frac{1}{4}$ *d.*				184,884375

8th. E X A M P L E.

If 40 Men are able to finish a piece of Work in 8 Dayes, how many may do the same in 5 Dayes? here, because the Answer ought to be greater than the singular Number 5, the last of the Relatives is the *Divisor*, and so you are free from the cumbersome Reflections, on a Reverse *Rule*.

5 *Dayes* ; 40 Men :: 8 *Days*
40
5) 320
Answer 64 Men

9. *EXAMPLE.*

If the *penny* Loaff, ought to VVeigh 18 *Ounces*, when the VVheat sells at 10 *ſh*. *Sterling. per Boll*, what ought the same to VVeigh, when the VVheat sells at 15 *ſh*. *per Boll*;

15 *ſh*. 10. *ſh*. :: 18 *Ounces*.
10 Answer
15)180 (12 *Ounces*
30

In Questions of 5 Numbers, you have, for the most part, two paires of Relatives, and but one singnlar Number, wherefore you may take any one of the paires of Relatives with the singular Number, to

find the first Answer, and that Answer will serve for a singular Number, for the other pair of Relatives in finding the last Answer.

9th. EXAMPLE.

At 5 *per Cent per Annum*, what will 197 *L*. 15 *ſh*. 8 *d*. come to in 7 Years? In this the Relatives are 100 *L*. and 197 *L*. 15 *ſh*. 8 *d*. for the first pair and, 1, Year and 7 Years for the 2*d*. pair, and 5 is the Singular.

L. Int. *L.* | *Ye. ; Ys. Interests.*
100 ; 5 :: 197,783 | 1 ; 7,75 :: 9,88916
5 | 7,75
The 1*st*. Ans. is 9,88916 *L. Inter.* | 4944582
69224166
692241666
766410416

So that the Answer is 76 *L*. 12 *ſh*. 9 ½ *d*. and very little more.

10th. EXAMPLE.

At 2 *Dollars per* L. *Flemiſh*, Exchange at 35 5/9 *sh*, *Flemish per L. St.* how many *L. Ster.* will 666 2/3 *Dollars* come to?

Doll.	L. *Fl.*	::	*Doll.*
25	1		666,666
			L. *Flem.*

25) 666,666 (266,666
166
166

In this *Division*, finding my 3*d. Dividual*, the same with the 2*d.* and, (because of the Reiterated Figures of the Dividend) understanding that it will allways be the same in *Infinitum*, I therefore Reiterate the 2*d.* of my Figures in the *Quotient*; to wit, 6, till I have five Figures in all, as the *Dividual-Point* Intimates: And seeing I have 3 *Decimal* in my Dividend, and but one in my *Divisor*, I understand that the last two, of my five Figures in the *Quotient*, must be *Decimals*, and the dashed Figure is added, because, (as hath been allready said) Infinites cannot be limit

limited under *Decimal* thirds; so that my 666 2/3 *Dollars*, makes just 266 L. 13 *sh.* 4 *d. Flem.* for my first Answer, and this must be one of the *Coefficients* for the 2*d.* operation, because in this we have now two Parcells of *Flemish Money*, and but one L. *Sterl.*

L. *Fl.*	L. *Sterl.*	L. *Flem.*	1 *sh.* 05
1,777	1	:: 266,666 1/3 *sh.*	,0055
		5 *sh.*	0277

1,777) 266,666 (150*l.St.* 1 L. 15 *sh* 1,175
1 L. 15 5 *sh.* 1,777
1777
8888
8888
00000

In this last Operation the third *Dividual. viz.* 0000 being less than the *Divisor*, I put 0 in my *Quotient*, and so the Answer is 150 Lib. *Ster.*

11*th.* E X A M P L E.

At 5 L. 10 *sh.* per *Cent per An.* what will the *Interest* of 756 L. 13 *sh.* 4*d.* amount to in 7 *Yeares*, and 7 *Months*?

L. 2 1 *Year*

Year L. Years Months 6=5
1 55 :: 7583 Months 1 =083
55 Months 7 =583
37916
L. Princ. 379166 Int. L. 100 for 1 Y.
100 41708 :: 756666 L. Pr.
75666
139027
139027
2502500 *Vide*. my *Compend*. system a-
20854166 nent *Multiplicati-*
29395833 *on*, with
31759305 L. In. an Infinite
in both *Coefficients*.

The Answer is 317 L. 11 *sh*. 10 $\frac{1}{2}$ *d*. very near

12*th*. EXAMPLE.

Interest at 5 $\frac{1}{2}$ *per Cent*. What will 456 L. 13 *sh*. 4 *d*. (payable 3 *Years* hence) be worth in present *Money*?

Years

Years L. Years.
1 5 55 :: 375
55
1875
1875
20625 L per.
Ce. (for $3\frac{3}{4}$ Y.

Add this *Interest* to its principal, and the 2 *d*. Operation will stand thus.

L Pr. & In L. Prin. Prin. & Int.
120625 100 :: 456666
100
45666666

120625) (45666666 (378583
947916
1035416
704166
Ans. 378 L. 11 *sh*. 1009416
8 *d*. & less than 444166
$\frac{1}{4}$ *d*. more. 82291

I shall conclude with an Example of unequall *Division*, which may be very useful in fellowship.

13*th*

13th. EXAMPLE

There is to be Divided amongst 14 Men 458 L. 5 *sh.* 11 *d.* with *Proviso*, that 9 of the Number (whose stocks or hazards were equall) have equall shares, but the other 5 are to have, one of them $\frac{1}{2}$ share another $\frac{1}{3}$ another $\frac{1}{4}$ another $\frac{1}{5}$ another $\frac{5}{6}$ of an equall share: The equall share, and consequently the severall Fractions of the equall share, is demanded?

In this you must add the *Decimals* of the severall Fractions in the Question to 9, and so you will find your *Divisor* to be 11,083 and not 14 thus

9000
$\frac{1}{2}$ = ,5
$\frac{1}{3}$ = ,33
$\frac{1}{6}$ = ,166
$\frac{5}{6}$ = ,83

The Sum of of all which is 11,083
Hence the Question must be thus stated.

Men	L.		Man	
11,083	458,2958	::	1	11083)

* L.

11083) 458,29583 (4135 to each of 9
4 × d = 4433333 20675 $\frac{1}{2}$ 20 L. 13 sh. 6
1496250 1378333 $\frac{1}{3}$ 13. 15. 8
1 × d = 1108333 103375 $\frac{1}{4}$ 10. 6. 9
387916 689166 $\frac{1}{6}$ 6. 17 10
3 × d = 332500 3445833 $\frac{5}{6}$ 34. 9. 2
55416. 37215 9 Quotie.
5 × d = 55416:458,2983
00000

If any Difficulty occur in these lessons, it may be easily overcome, and at a very Reasonable Rate, by a little converse with the AUTHOR.

FINIS.

ERRATA. *Page 29, Line 4. read* 8 *for* 6. *P.* 32. *L.* 8. *R.* 9276. *for* 8276. *In the end of the Product. also the Example Page.* 38. *is only disorderly set* *r the* 9 *of the Quotient ought to stand under the* 2d. *of the Dividend. and the rest in Order.*

At *Edinburgh*, the Twentie eight Day of *November*, *one Thousand*, *six Hundred Nintie Nine Years*.

THe Lords of His Majesties *Privie Council* Having by their Act of the first of *December*, *one Thousand*, *six Hundred*, *Nintie Eight Years*, Granted to Mr. *George Brown* Minister, and his Heirs and Assigneis, the sole Priviledge of framing, making and selling his Instrument called *ROTULA ARITHMETICA*; For the Space of fourten *Years*: From the *Day* and *Date* of the said Act. And Discharged any other Persons to make and sell the said Instrument, during the Space foresaid without express Liberty and Licence, from the said Mr: *George* and his foresaids: under the Paine of five Hundred Merks; besides the Confiscation of the *ROTULA's*, Made or Sold. The said Lords of his Majesties *Privie Council* Do hereby Discharge the Importing of the said Instrument, or *ROTULA*; During the Space foresaid, alse well as the making or Selling thereof: Under the said Paine of Five Hundred Merks: Besides the Confiscation of the Instruments or *ROTULA'S* Imported. Alse well as these made or Sold. And declares, this present Act, to the Commence from the *Date* of the former Act: Which is the first *Day* of *December*: *One Thousand*, *six Hundred*, *Nintie Eight Years*.

Extracted by Me,
Sic subscribitur,
Gilb. Eliot Cls. Sti. Cons.

THE Principles of Geometrie,

Astronomie, and Geographie.

Wherein is breefely, euidently, and methodically deliuered, whatsoeuer appertaineth vnto the knowledge of the said Sciences.

Gathered out of the Tables of the *Astronomicall institutions of Georgius Henischius.*

By *Francis Cooke.*

Appointed publiquelye to be read in *the Staplers Chappell at Leaden hall, by* the Wor. *Tho. Hood*, Mathematicall Lecturer of the Cittie of London.

AT LONDON
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Printed by *Iohn Windet*, and are to be solde in Mark lane, ouer against the signe of the red Harrow, at the house of *Francis Cooke.*

To the VVorſhipful wel-willer and practiſer of the Mathematicks, M. *Bernard Dewhurſt*.

YOur courteſies (vvorſhipful) inexpected, haue brought me very farre in arrerage with you, and I haue thought the time ouer long, vntill in ſome ſort I might break ſilence, and ſay ſo. But better late then neuer, as the prouerb goeth, and yet perhaps as good not at al, as not as it ſhould be. I haue intẽded many waies according to the courage of a ſcholler, to be euen with you, or at leaſt to ſhew ſome ſign of gratitude, but it would not be: and therefore haue I reſted ſo long vpon hope of ſome fit oportunitie, which now being fitly offered, I haue accordingly taken holde of: I preſent therfore this little booke vnto your Wor. little I meane in bodie, but full of ſubſtance and matter: according to the commendacion which the Poet giueth to *Tydeus*, the Father of *Diomedes*, and that was that *Tydeus*, *Ingenio magnus, corpore paruus erat.* A little fellowe, but full of edge. I commend it vnto your Wor. although the praise therof shall rather proceede from it selfe, then from me, vnlesse I could praise it worthily enough. I found it els where, and otherwise attired, and my labour and charge is onlie in this, that I haue bestowed thereon a new coate after our English fashion, obseruing the matter, only altering the manner. Accept it I pray you none otherwise then I mean it, not as a guift worthy enough, but as some little signe of a thankfull minde, which according as power shalbe answerable vnto any good occasion, I will more manifestly declare in some greater matter.

Your Worships wholy.
F. Cook.

To the learned and worshipfull. *Tho. Hood* Mathematicall Lecturer *of the Cittie of London.*

AMong many your schollers whose diligence haue made them more able: I haue aduětured to put forward my selfe, being of all other most vnfit and insufficient. My purpose herein is not the gaining of mine owne commendation, but to cōmunicate that with others, which I finde beneficiall vnto my selfe. The commendation is wholy yours, from whome as from the Sunne we receaue, as in these exercises, all the light we haue. And hereunto may be added another cause of this my bold enterprise, namely, that others, who are better furnished, may by mine example, being a nouice in these studies, make proofe vnto the worlde, that your labours are not in vaine, but that there are many which haue greatly profited thereby, and which in due time (I hope) will breake forth, both to the great commoditie of the common wealth, and commendation and credit of the Mathematicall Lecture. Take in good parte (I pray you) these my first fruites of your owne planting, and make fauourable construction thereof, correcting the faults, and pardoning my boldenes.

Your Scholler in this case more forward then able. F. Cook.

To the louing and diligent Auditors of the Mathematicall *Lecture.*

GEntlemen and Goodfellowes, and to ioyne you both in one, men and schoolfellowes: I haue as you see aduentured to flie, although both fitted with the feathers of another, and lying but a lowe pitch, for want of winges. If I shall seeme to any of you too forward, I craue his pardon, it is my first fault, I meant no hurt: If any man will charge me of defects, I will confesse my wants, and submit my selfe vnto any reasonable censure. Howbeit I look for no hard measure from any, frequenting these exercises, either in the schoole or abroad, inasmuch as they are for the most parte tame beastes that belong to this folde: what the wilde and sauage sorte can or will doo I feare not, I can appeale from them, as in this case, no competent Iudges. I haue after my manner dealt plainely with the matter, and I pray you take it as it is, inasmuch as howsoeuer it deserueth, I can set it out no better. For as one saith.

They that are learned and haue the gift,
may make of matters what they will,
But he that hath none other shift,
must goe the plaine way to the mill.

As in this I haue doone,
Fare you well.

The

¶ The descriptions of Geometricall *and Astronomicall termes.*

Of Magnitude. Chap. 1.

THere be 3. kindes of Magnitudes, a line, a surface, a body.

In a line, called by the Greekes *γραμμὴ*, we are to consider the definition, the termes, the sortes.

The definition of a line is two folde: For it is defined to be either a length without breadth, *μῆκος ἀπλατὲς*, or els the flowing of a point in length.

The tearmes of a line are points: and here we are also to note, the distinction of pointes, and their denomination.

They are distinguished into points Geometricall, and Physicall.

Geometricall points are such as haue no part.

Physicall points be such as may be apprehended by our sight, of which sort are motes in the Sunne shine.

Pointes are denominated either centres and poles, whereof the principall are those of the worlde, of the Zodiake and of the Horizon: or els the Equinoctiall, and the Solstitiall pointes.

For the seuerall sorts of lines look in the 2. Chapter.

In a Surface, in Greek *ἐπιφάνεια* we are to attend the definition, and the termes.

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broad extremitie, or els length, and breadth without depth.

The termes of a ſurface are lines, whereof in the Chapter following: as alſo of the ſeuerall ſorts of ſurfaces in the 3. Chapter.

A bodie is defined to be breadth, hauing depth adioyned therunto: or a figure conſiſting of 3. dimenſions.

The bounds or limits of a bodie are ſurfaces.

There be many ſorts of bodies: wherof in the latter parte of the book.

Of Lines. Chap. 2.

A Line is either right or crooked.

In a right line note, the definition, and the diuers kindes therof.

A right line is defined to be, either the ſhorteſt extenſion from one point vnto another: or els, the ſhorteſt of thoſe lines that haue the ſame limits.

There are 4. ſortes of right lines.

The firſt diuideth a figure into 2. equall or vnequall partes.

Right lines diuiding a figure into 2. equall partes, are either diameters, namely in a ſquare, or in a circle: or Axes, as in a Spheare: or els diagonales, as in *Polygons.*

Right lines diuide a figure into 2. inequall partes, as chordes in circles, the halfes wherof are called Sines.

The ſecond ſorte of right lines are thoſe that bound the figure, whereof thoſe that limit the

vpper

vpper parte, are called *Corauſci*: thoſe that boũd the nether parte, are called the *Baſes*: thoſe that include the laterall partes, are called *Coſtæ*, the ſide lines.

The third ſort of right lines are ſuch, as are eleuated either perpendicularly, or not perpendicularly.

Right lines perpendicularly eleuated, are called perpendiculars, plumb lines, ſquire lines, orthogonall lines, or lines at right angles.

Right lines not perpendicularly eleuated, are called Hypothenuſailes or ſubtendent lines, alſo oblique lines: likewiſe viſuall lines, or viſuall beames.

The fourth ſorte of right lines, are ſuch as are equidiſtant, as paralleles.

In a crooked line conſider the definition, and the ſortes therof

It is defined to be the running of a point vnto a point, not by the ſhorteſt, but by a longer way.

The ſortes thereof are many: whereof ſome are ſimple, and ſome mixt.

Simple crooked lines are ſuch as are made by a point running round, as in the circumference of a circle, of a ſemicircle, and of an arke of a circle.

Mixt crooked lines are ſuch as compaſſe not a circle: and they are either winding, or ſpirall.

Winding lines are ſuch as in their partes are inequallie eleuated from the midſt.

Spirall lines are ſuch as being wound round about either ſome centre in a place ſurface, or ſome piller, as we may ſee in cylinders and the

ſcrues

serues of presses, do neuer returne vnto the same point from whence they began.

Of Surfaces. Chap. 3.

SVrfaces are either plane, or Sphericall.

Plane surfaces are such as are limited equally either with right, or els with crooked lines.

Plane surfaces limited equally with right lines, are such as haue either 3. or 4. or more right lines for their limits.

Plane surfaces equally limited with 3. right lines, are triangles, which haue their denominations either from the lines that inclose them, and then they are saide to be of equall sides, of equall feete, of inequall sides and feete: or els from their angles, and so they are said to be right angled, obtuse angled, acute angled.

Plane surfaces equally limited with 4. right lines, are either parallelogrammes, whereof some are squares, some long quadrāgles, some Rhombes, some Rhomboides: or els, they are Trapezia, or Tables.

Plane surfaces equally limited with more thē foure right lines, are of many sortes, as a Pentagon, an Hexagon, &c.

Plane surfaces equally limited with crooked lines, are circles, whose partes are called arkes, portions, or sections, a semicircle, a quadrant, a segment, or a sector of a circle.

Arkes are partes of the circumference of a circle

le separated by chords.

Portions or sections are the greater and lesse surface of the circle, distinguished by a chord.

A semicircle is that which is conteined vnder the halfe circumference, and the Diameter.

A quadrant is the fourth parte of a circle, included within 2. semidiameters.

A Segment or sector of a circle, is a figure conteined vnder an arke of the circumference, and 2. right lines drawen from the centre.

Sphericall surfaces are such, as are limited and conteined vnder inequall, that is, vnder depressed and eleuated lines.

Sphericall surfaces are either conuexe, or concaue.

Conuexe sphericall surfaces are such, as doe bound the body on the outward parte.

Concaue sphericall surfaces are such, as do limit the body on the inward side.

Of Angles. Chap. 4.

AN Angle is made by the alternate or crosse meeting of lines or surfaces.

An Angle is either superficiall, or solide.

In a superficiall angle there are to be considered, the definition, and the diuision.

A superficiall angle is defined to be the touch of two lines, the one inclining to the other in one surface.

Superficiall angles are diuided two waies, for they are considered either by them selues, or relatiuely.

Superficiall angles considered by themselues, are either plane, or sphericall.

Plane superficiall angles are such as are drawen vpon a plane surface, and they are either right lined, or crooked lined, or mixt.

Right lined plane superficiall angles are such as are made by right lines onely, and those lines are either perpendiculare, wherof a right angle is made, which is alwaies equall vnto the right angle next adioyning vnto it selfe: or not perpendiculars, wherof is made either an acute angle, lesse then a right angle, or an obtuse angle, greater then a right angle.

Crooked lined plane superficiall angles are such, as are made of crooked or bowed lines onely.

Mixt plane superficial angles, are such, as one right and one crooked line doth make.

Sphericall superficiall angles, are such as are drawen vpon the conuexe surface of the sphere, differing according to the compasse of the greatest circle, described from the very toppe of the section.

Superficiall angles are considered relatiuelye when they are compared with others, in whose respect they are called ioynt angles, verticall, alternate, and opposite angles.

Ioint angles are such, as a right line falling vpon a right line, maketh on either side.

Verticall angles are such, as the mutuall ioyning together of two lines, doth make on contrary partes.

Alternate angles are such as one line falling vpon 1. dooth make both on the right and lefte hand of either of them, aswell within as without.

Opposite angles are those that can haue no relation vnto any one of the former.

A solide angle is that which is considered in solide bodies, contened vnder more then two plane angles, not situated in one and the selfe same surface.

Of Bodies. Chap. 5.

THe kindes of the third magnitude which is called a bodie, are diuerse: some are regulare, others are Irregulare.

Regulare bodies are such as are limited by equal surfaces, the which surfaces are either turned round, or foulded one toward another.

The surfaces turned round, are either the equall sections of circles, or right lined figures.

Equall sections of circles, are such as either fil vp the whole plane, and by them the spheres are made, or els they are cut out hollowe in the midst, wherof are made orbes, either vniformed or difformed.

Right lined figures limiting regulare bodies, are either right angled triangles, wherof Pyramides are made, whose vpper parte is called the Cone, or toppe, and the nether parte and plane surface is called the base, or square or quadrangulare figures on the one side longer, from whence are deriued figures long and broade, as pillers, or cylinders: or els they are of other sortes,

tes,which are infinite, from whence diuers formes and kindes of bodies are drawen.

The regulare bodies conteined vnder surfaces, folded one toward another, are onely these 5. the Tetraedrons, the Hexaedrons, the Octaedrons, the Dodecaedrons, and the Icosaedrons.

Irregulare bodies are such, as inequall surfaces do limit and describe, the which surfaces are either turned roũd, or folded one toward another.

The surfaces turned round and making irregulare bodies, are either the sections of circles, or els they are inequall right lined figures.

The sections of circles, are either greater then a semicircle, wherof the lenticulare bodies are made: or els they are lesse then a semicircle, and therby are the Oualles made.

The inequall right lined figures by whose cõuersion the irregulare bodies are made, may be of what sorte soeuer, wherby diuers kindes of vessels are framed, either wanting or exceeding the regulare forme.

The Irregulare bodies made of inequall surfaces folded one toward another, may differ infinitely.

Of the name and definition of the Sphere. Chap. 6.

IN as much as we make often mention of the Sphere, and thereafter do intitle this present treatise the doctrine of the Sphere, it shall not be amisse to declare the name, and the definition therof.

The name is vsed in diuers significations.

1. It

1. It signifieth sometime any solide regulare bodie, limited with one surface onely.

2. Sometimes it signifieth an instrument that doth instifie the apparences of heauen, and conteineth the celestial circles, and is otherwise termed a ring or materiall sphere.

3 Sometimes it signifieth the whole worlde, wherunto all the cõditions of the sphere may be applied: For it is a solide bodie, wherin nature abhorreth that any emptines should be giuẽ: It hath a sphericall forme running dayly about his owne Axis without intermission: It hath a point placed in the midst therof, namely the earth.

The definition thereof, as it signifieth any body, is by *Io. de sacro bosco*, set downe two waies, the one after *Eucl.* 11. *Elem.* the other out of *Theodosius.*

The definition thereof taken out of *Euclide*, conteineth the Geometricall description of the sphere: For the sphere is described by the fixed and vnmoued diameter, and by the arke of the semicircle, which must be fully brought about.

The definition of a sphere according to *Theodosius*, determineth, first the orbiculare forme, euery parte whereof is equally distant from the centre: secondly the principall partes, as the conuexe surface which is but one, and the centre, that is, the point in the midst equidistant from euery parte of the surface, and the Axis, about which the sphere is tourned, and which is limited by the 2 poles. viz. The North pole or pole Arctick, and the south pole or pole Antarcticke: and thirdly, the soliditie: For it is a complet

plet

plete body, hauing all the dimensions.

The diuision of the celestiall Sphere. Chap. 7.

THe celestiall sphere, according to *Io. de sacro bosco*, admitteth a double diuision, according to substance and according to accident.

The celestiall sphere considered according to the substance, is diuided into seuerall orbes, in the which we are to note the number, and the cause.

The number is diuersly set downe: For the an-cientes contenting themselues with 8. orbes on-ly, did distinguish them into the orbes of the 7. Planets, viz. of ♄ ♃ ♂ ☉ ♀ ☿ and ☽, and the fir-mament of the fixed starres: And the later A-stronomers vnto the time of *Alphonsus*, into 9. orbes: but the Moderne, among whom *Purba-chius* was the first, added the tenth.

The cause is considered either in the diuersitie of their number, noted both by the former and later Astronomers, or in their order.

The ancient Astronomers noted their diuers number, either by the brightnes of the Starres, reconing so many orbes as they perceiued to conteine any starres: or by the peculiar motion of each seuerall orbe, reconing so many orbes as they found simple motions belonging there-unto.

The later Astronomers, for instruction and the better reconings sake, added the ninth and the tenth, as circles necessarye for the vnderstanding of the motion of the 8. sphere, vnknowne vnto

the ancient Astronomers.

The order is proued, 1. by the slower motion of the higher, and the swifter motion of the low-er orbes: 2. by the occultation or hiding of the higher starres, by the lower: 3. by the diuersitie of apect, either great, or little, or insensible.

The celestial sphere considered according vn-to accident, that is, according to the situation of the heauen, or the course of the starres, is di-stinguished into a right, or an oblique sphere.

The right sphere belongeth vnto those that dwell vnder the Equinoctiall, who (by reason that the poles of the world, about the which the starres are carried by the firste moueable, haue none eleuation, as also for that the Horizon cutteth all the paralleles, vnder which the stars do goe, at right angles) perceiue no reflexion in the diurnall motion of the starres.

The sphere is said to be oblique, wherein the ☉ and the rest of the starres are caryed from the East into the West, by an oblique motion, and it is Septentrionall vnto those that haue the North pole eleuated, and Meridionall vnto those vnder the Southerne eleuation.

The partition of the whole worlde: and the comparison of the celestiall, with the Elementall Sphere. Chap. 8.

THe whole frame of the worlde is made of some certain and those more principall and notable partes, wherof there is first the number and the name, and then the difference to be considered.

The number and name is double: For the partes are either ethereall, or sublunare.

The æthereall, that is, the celestial parte (without the which Philosophie admitteth nothing to be, although the Diuines do adde the third, which they call Angelicall, and the Platonickes, intellectuall) is that, wherof we intreated in the 7. Chapter.

The sublunare is that which conteineth the elementall bodies, and those either simple, as the fire, the aer, the water, the earth: or els mixed, which are diuerse and innumerable, ingendred of the 4. elementes, either perfect or imperfect.

The difference or dissimilitude of the partes of the worlde is that, whereby they are distinguished one from another, either in respect of their situation, or of their dignitie, magnitude, motion, or their office.

They are distinguished according to their situation: For the celestiall parte hath obteined the higher place, the elementall the lower.

Their distinction according to dignitie, is noted in the partes conteined by the celestiall Region, which partes are bright and immortall, and by the elementall region, those partes being of their owne nature obscure and decaying: or els, it is noted in the partes conteyning, wherof the one is altogether without alteration, neither increasing nor diminishing, the other is continually subiect vnto generation and corruption, and

and is increased and diminished.

Their distinction according to their magnitude is considered, in that the celestiall parte with the great compasse thereof, doth couer all thinges, like a thing without measure and ende: but the elementall parte is couered within the compasse of the heauen, the diameter thereof conteining the diameter of the earth, 23. times.

Their distinction according to their motion, is in that the celestiall parte hath a circulare, and a sphericall motion: the elementall, a right motion, more imperfect then the circulare.

Finally, they are distinguished according to their office: For of those thinges that are ingendred in the elementall parte, the heauen, working by a continuall motion, is as it were the formall and efficient cause, from whence life is deriued: and the elementall parte, which is subiect vnto passion and alteration, is as it were the materiall cause, from whence nourishment doth proceede.

The reason of the sublunare, or elementall Region. Chap. 9.

THe Elementall region, which the heauen encompasseth, comprehendeth within it the elements, wherin we are to consider the definition, the number, and the situation or order.

The elements are simple bodies, aswell in respect of the mixt bodies which are vnderstoode to be compounded of them, as of the simple and least partes: as also in respect of the diuision, for that

that they cannot be diuided into bodies of diuers kindes (if they be giuen pure and without mixture. For the vse of liuing creatures, and things growing doth make them impure.)

The elements are 4. in number, found so to be, by sense, and by reasons,

The elements are found to be foure by sense (which the Physicians doe follow): First for that more simple bodies cannot be shewed: 2, nature hath alotted vnto them certaine places, to the end that other things might by thē be bred, and nourished: 3, nothing els can euidently be shewed, wherof other things may be made: 4, in liuing cretures there are certain parts, agreable vnto the natures of the seuerall elements.

The Elementes are found to be foure, by two reasons: the former whereof is drawen from the number of the foure prime qualities, and the foure folde possible knitting together of them. For heate may be ioyned either with drinesse, which two make fire, or els with moisture, which two do make vp aer: and colde may be ioyned with moisture, as it commeth to passe in the water: or with drinesse, as in earth. The later reason is taken from the fowerfolde difference of the right motion: For the elements are directly moued, either vpwarde or downward.

Such things as moue vpward as light thinges do, are said so to do, either simplie, as the fire, which is the lightest of the rest: or respectiuelie, as the aer, which is lighter then the water, or the earth.

Such

Such things as moue downeward, as heauye things do, are said so to do, either respectiuelie, as the water compared vnto the fire, and aer: or simplie as the earth, which is the heauiest of all the rest.

The situation and order of the Elementes, is found either by their motion, or els by the communication of their qualities.

And first by the motion: For inasmuch as the fire and the aer do naturally moue vpward, the fier shall occupy the highest place: the aer, an vpper place: and for that the water and the earth do naturally moue downward, thē water shall possesse a lower place, and the earth the lowest.

Againe, the order of the Elements is found out by the communication of their qualities, for it were vnfit that such things as are merely contrary, but such as in some sorte can agree together, should be nigh one another. The fire therefore shalbe ioyned vnto the aer, by reason of the heate common vnto them both: the aer vnto the water, by reason of the common moisture: and the earth vnto the water, by reason of coldnesse common to them both.

III

The two folde differences, of the celestiall motions. Chap. 10.

THe whole frame of the world is caried round about, with 2. motions, each of them being distinguished from the other in name, and in reason.

The one of them is called the first and vniuer-

sall

sall motion : likewise the diurnall or worldlye
motion, because it bringeth the day vnto the
world. For in this motion the ☉ and all the ce-
lestiall bodies do euery day arise and set : they
call it also the violent, and rapt motion, because
by the violent swiftnes thereof, it carrieth with
it the rest of the Spheres.

The other is called the second and particular
motion, altogether contrary vnto the former, as
by which all the particular orbes do resist the v-
niuersall motion. They call it also *Sinister motus*,
the motion to the left hand, as the former is in
like sorte called *dexter*, that is, the motion to the
right hand.

The 2. motions are also distinguished accor-
ding to the reason or the substance in the which
they are inherent: For they differ the one from
the other three waies.

The first difference is in respect either of the
whole : For the diurnal motion is common vn-
to all the celestiall bodies : or els of the partes,
or starres either fixed or wandring, which haue
a motion peculiar and propre vnto themselues.

The second difference is either in regarde of
the situation of the Axes, For the diurnall mo-
tion is made vpon the Axe and poles of the
world, and therfore the Equator diuideth it in
the middle: but the propre motion is made vpon
the Axe and poles of the Zodiake, and therefore
the Zodiake doth cut it in the middle : Or els it
is in regarde of the position of the termes: inas-
much as the diurnal reuolution is made from
the East vnto the west, or as *Plinie* termeth it,
from

rom the right towarde the lefte hand: but the
propre reuolution is from the West vnto the
East, or from the left toward the right hand.

The third difference is in consideration of the
swiftnes : For the diurnall motion fulfilleth his
course within the space of 24. common howers:
but the propre motion in diuers distances of
time, according to the largenes of the orbes :
namely, the orbe of the fixed starres performeth
his circle, in 36000. yeares : of ♄ in 30: of ♃ in
12 : of ♂ in 2. yeares: of the ☉ in 365. dayes,
and about 6. houres : of ♀ in 384. dayes, after
Plinie: the orbe of ☿ in as many dayes as the ☉:
and the orbe of the ☽, in 27. daies & 8. howres.

The circulare forme, and circulare motion of the heauen. Chap. 11.

THe Heauen is circulare in motion, and in
figure.

The circulare motion of the Heauen is pro-
ued as well by 2. experiments, as by 2. argu-
mentes.

The one experiment is taken from the starres
of the 8. orbe, which both in their rising & set-
ting, do alwaies keepe one & the same habitude,
both in regarde of the earth, and one to anoth r:
which thing can agree with none other then a
circulare motion about the centre.

The other experiment is also taken from the
starres of the 8. orbe, alwaies appearing, and re-
taining in diuers places the same distance from
the

the Poles, and one from another: which also agreeth with the circulare motion onelie.

The first argument is deriued from the confutation of 2. opinions, wherof the one supposeth that the motion of the heauen is direct and infinite: which if it were, the starres should vanishe out of our sight: The other, that the starres in their setting are quenched, and in their rising are lighted againe: as *Heraclitus* affirmed, which is absurd, both in respect of the motion, which is perpetuall and constant in it selfe: and of the contrary power, which cannot be in the earth: as also in regarde of our Antipodes, whose West is our East.

The second argument is drawen from the dignitie thereof: For the circulare motion is the most worthye, and more perfect then the right motion, inasmuch as it breedeth no scission or cutting, and is made about the midst of the whole, not by displacing the whole bodie, but by the onlie vnchangeable succession of the situation of the partes.

The criculare figure of the Heauen is proued partely by similitude, and partely by reasons.

The similitude is this: namely that this sensible worlde is the image of the first Archetype, or paterne of the worlde, who is without beginning or end.

The reasons do containe either the commoditie of the circulare figure, or the necessitie.

The commoditie consisteth either in the capacitie, or els in the swiftnes or aptnes vnto motion.

The

The capacitie was fit for the heauens, in that they were to comprehend all other things. For the circulare figure is the greatest of al other, circumscribed with equall conuexities.

The swiftnes or aptnes vnto motion, is either belonging vnto the diurnall motion, called also the right hand motion, naturall vnto the Heauens: or els vnto the second motion, which resisteth the former.

The reasons drawen from the necessitie of the circulare figure, are either in respect of the whole world: For if the Heauen were of any other figure, there must needes be some emptie place, and a body without a place: or els in regarde of the celestiall orbes, which either could not be turned about by diuerse motions, or els they should suffer a scission or cutting in sunder not without their greate hurt.

There is one surface of the earth and water, and that is round. Chap. 12.

THe earth and the water make one globe, and it is proued by causes either generall, or speciall.

The generall causes belong vnto both the elements made vp in one forme, and are deriued from 3. heades.

First from the signification of the worde: For both in common speach, and in the scriptures, it is called *Orbis terræ*, the Globe of the earth, or the round world.

Secondly, from the Sphericall forme aswell of the

the Heauen, into whose round compasse inasmuch as it is included, it cannot be but it must also be round: as also of the shadowe that this globe doth cast forth: For the Maisters of Perspectiue do teach vs, that such is the darke bodie as is the shadowe therof.

Thirdlye, from the naturall descending of the portions, either of the earth, the saide portions coueting the centre of the Globe, and falling vpon the surface of the earth at right angles: or els of the water, sinking also into the centre of the worlde, for the which their discending they gather themselues into a round forme, and cannot abide vpon a plane surface.

The speciall causes are such, as decerne the roundnes of the earth, and of the water particularely.

The roundnes of the earth is decerned two waies.

The one is according vnto Longitude, from the East toward the West, or contrarywise, and that either by all the starres, which in diuers places do not appeare at the same instant: or els, cheefely by the ☽, whose Eclipse falleth out at one and the same time, but by those in the East reconed one way, by those in the West, another.

The other is according vnto Latitude, from the Equator towarde each pole, gathered by the vnlike eleuation of the Pole, and inequall quantitie of the daies, both which increase vnto those that goe from the Equator towardes the North or South,

The

The roundnes of the water is decerned by tokens deriued from the swelling of the droppes, either hanging, or throwen vpon the dust, or laide vpon the mosse of boughes: as also from the swelling of the Sea, by meanes whereof the Land cannot be seene from the Shippe belowe, although from the maine toppe it may: and againe if any shining thing be fastened to the top of a ship sayling farre from the shore, it descendeth by little and little, according as the Shippe runneth further off, and at the last is hidden from the sight.

The situation, immobilitie, and magnitude of the terrestriall Globe. Chap. 13.

THe earth or globe of the earth and water, hath situation, rest, and magnitude.

The situation as being in the centre, or the place of the worlde farthest distant from the extremities thereof, is proued by arguments either direct, or indirect.

The direct are deriued from the nature of the broken partes, expressed either Physicallye, or Astronomically.

Physically, because wheresoeuer they are about the earth, we alwaies obserue them, that of their owne inclination they tende downewarde. But the centre is the lowest place.

Astronomically, inasmuch as all the semidiameters of the worlde, by which heauy thinges descende, are continued through the centre of the worlde, and there they cut one another. So that

that where the section is made, there must needs be the place of the earth.

The indirect arguments consist in two suppositions.

The one, that the earth were in the Axis of the motion of the Heauen towarde one side, & then should be taken away the apparent reason of the middle Heauen: the reason of the shadowes Equinoctiall, Solstitiall, and plagall: and the reason of the vniuersall Equinoctialles.

The other supposition, that the earth were without the Axis, remoued from the poles, either to the East or to the Westwarde, and then shalbe taken away, both in the rising and setting, the equall quantitie both of the daies, shadowes, and starres.

The Rest of the earth excludeth al locall motion, either right or circulate.

The right motion is that which is made from the midst, and it is either naturall and peculiar vnto the earth: For otherwise it should come to passe that heauy things should ascend: or els it is violent, some outward thing inforcing it: For otherwise it should come to passe, that the earth should forsake the centre of the world.

The earth hath no circulare motion, neither from the West to the Eastwarde, as some haue thought: For if it had, all things that are moued in the aer, should alwaies be moued to the westward: Neither from the East to the Westward, by the diurnall motion: For then it should be an harder matter to trauaile toward the East, then toward the west.

The

The magnitude of the earth is nothing, being compared either with the whole world, wherof it is the centre, which is proued by Mathematicall instrumentes that agree with the centre of the world: For they at one time, and through the same sight hole, shew two Starres opposite in the Diameter: or els being compared but with the orbe of the ☉, which is proued by the equall spaces of the dayes and nights.

The measure of the compasse of the earth. Chap. 14.

THe circūference of the Globe of the Earth and water is found out by the rule of foure proportionall numbers, in which rule 3. numbers are giuen, and the fourth is vnknowne.

The three numbers giuen, which conteine the proportion of a segment of a celestiall circle vnto the like space on the earth, are: 1. the difference of Latitude: 2. the viatorie distance: 3. the circumference of the whole heauen.

By the difference of Latitude is vnderstoode, so many celestiall degrees, as any terrestrial places are distant asunder.

The viatorie distance, is that terrestriall space of waye, that is answerable vnto one degree, or any other difference of Latitude, and it is found out 3 manner of wayes.

First, by the distance of any two places vpon the earth, situated vnder one meridian, the said distance being precisely tryed.

Secondly, by the latitude of both places, either

ther obserued by instruments, or taken out of Tables.

Thirdlye, by subducting the lesse out of the greater: for so the difference of latitude shall appeare, whereunto the space of way knowen betweene the places geuen, shalbe answerable. Whereby vnto each degree of a great circle in the heauen, there are answerable vpon the earth after *Ptolemee*, furlongs 500. passes 62500. greater leagues 15. After *Eratosthenes*, furl. 700. pass. 87500 leag. 21. $\frac{7}{8}$.

The circumference of the whole heauen (conteining 360. gr.) is the 3. number in the proportion: 1, for that of a little and of a great globe, there is the like reason: 2. because the terrestriall meridian hath the same centre with the celestiall.

The fourth number of the proportion, that is the circuit of the greatest circle in the earth, hath 2. considerations.

The first is the maner of the searching thereof, and that is, first by multiplying the third number, that is, the circumference of the heauen by the second, which conteineth the space of way vpon earth: and then by diuiding the product by the first, which is the difference of latitude.

The second consideration is of the quotient, or manifestation of the content, which according to *Ptolemee* is miles 22550. furl. 180000. pas. 22500000. greater leagues, 5400. according to *Eratosthenes*, miles 61250. furl. 252000. pass. 61250000. greater leagues 7875.

The

The measure of the Diameter, and Semidiameter of the earth, as also of the Area and Surface thereof. Chap. 15.

IN measuring the terrestriall Globe, wee consider either the Diameter, or the Semidiameter, or the Area, or the conuex surface thereof.

The Diameter is measured by the proportion thereof vnto the whole circumference, by the rule of foure proportionall numbers, wherein againe three are geuen, and the fourth is vnknowen.

The three numbers geuen, beeing throughlye knowen and vnderstood, must be duely placed, and they must conteine two things.

The first is, the proportion of the circumference of a circle vnto the Diameter thereof, which is *tripla sesquiseptima*: that is to say the circumference is vnto the Diameter, as 22. is vnto 7.

The second is the greatest circuit of the earth in any measure, which was set downe in the 14. Chapter.

The fourth number of the proportion being vnknowen, is the Diameter, which is sought, first by multiplying the thirde by the second, which is 7. & diuiding the product by the first, which is 22. then by subducting the 22. parte (which commeth forth of the diuision of the circumference by 22.) out of the circumference, & diuiding the remainder by 3. whereupon ariseth the content of the Diameter, after *Ptolemee* conteining

teining miles 7159 $\frac{1}{11}$. furl. 57272 $\frac{8}{11}$. pass 7159090 $\frac{10}{11}$. greater leag 1718 $\frac{2}{11}$. After *Eratosth.* miles 19488 $\frac{7}{11}$ fur. 80181 $\frac{9}{11}$. pass 1948863 $\frac{7}{11}$. Greater leag. 2505 $\frac{1}{2}$.

The Semidiameter is the distance between the conuex surface of the earth, and the centre thereof (which some do imagine to bee the place of hell) the said distance is found two waies.

1 By the proportion of the circle vnto the Semidiameter, which is *sextuple* ouer and beside $\frac{1}{14}$: or as 44 is vnto 7.

2 By diuiding the Diameter into two parts: by which meanes it shall bee found to conteine after *Ptolomee*, miles 3579 $\frac{1}{2}$. furl 28636 passes, 3579545 greater leag. 859 $\frac{1}{11}$. After *Eratosthenes*, miles 9744. furl 40090 $\frac{1}{2}$. pass. 974431. greater leag. 1252 $\frac{1}{2}$.

The Area or plane is founde by multiplying halfe the circuit of the earth taken in anye knowen measure, by the Semidiameter thereof.

The conuex surface that couereth the whole earth, is founde by multiplying the terrestriall Area or plane, by 4.

The generall definition and diuision of the circles. Chap. 16.

IN as much as the surface of the Heauens is sphericall, and their motion circulare, therfore for the better conceiuing of the reasons of the celestial motiõs, they are distinguished into certaine circles as partes, whereof we are to shew the names & the diuision.

In

In the names we are to consider their acception, and their diuersity, being notwithstanding all one in signification.

The acception of the name Circle, is of two sortes, Geometricall, and Astronomicall.

The Geometricall acception, is when a circle is taken for a plane figure, which one line equally distant from the centre doth encompasse.

The Astronomicall acception is either as it signifieth a circular line, or a circumference wanting breadth: or else a circulare surface, which hath breadth therunto adioyned.

The diuersitie of names all one in meaning, is when circles are called (amongest diuers Authors) threds, compasses, orbs, segments, rings, paralleles & equidistant lines.

The diuision of circles is diuersly deliuered by the Greekes and Latines, in three respectes. First in respect of the materiall sphere, within the which some of the circles are not placed, & are therefore called extrinsecal, fixed, and manifold, as the Horizons and the Meridians: others are placed within the sphere, & are therfore called intrinsecall, moueable, and singular, as are the two polares, the Equator, the zodiak, the two colures, and the two tropickes.

Secondly, in respect of the poles of the world, or the twofolde motion of the heauen: and in this case the Greekes distinguish them againe into three sortes.

The first are paralleles, in number 5. namely the 2. polares, the 2. tropicks, and the Equator, all which haue the same poles with the world,

are

are equidistant on all sides, and serue the fyrst or vniuersall motion.

The second are oblique circles, in number 3. namely the Zodiak seruing the second motion, the Horizon, the milke way, all the which lye oblique betweene the poles.

The thirde are those circles that are drawen through the poles, and they are also 3. in number, namely the Equinoctiall and solstitiall colure, and the Meridian

The thirde diuision of circles is in respecte of their quantity, according whereunto some circles are called greater, and some lesse.

The greater circles are in number 6. namelye the Equator, the Zodiak, the 2. Colures the Horizon and the Meridian, all which are equall one vnto another, and cut the sphere into equall pieces.

The lesse circles are in number 4. namely the 2. polares, and the 2. tropicks, which are not all of them equall one vnto another, neither diuide they the sphere into two equall pieces.

Of the greatest circle conteining the measure of the first motion. Chap. 17.

THe whole heauen or vniuersall frame, turned round by the first motion, doth in the middle place betweene the 2. poles, describe a certain circulare compasse, whereof we are to consider the name, the definition, the commoditye.

The name thereof is diuers: for it is sometimes called

called ἰσημερινὸς, *Aequidialis*, that is, if we may so tearme it, equidiall, sometimes the line or the orbe of the equalitye or equation of the day: sometimes the Equinoctiall and Equator: and sometimes the girdle of the first motion or moueable.

The definition thereof is that, wherein the magnitude, situation, and equall conuersion thereof are conteined.

The magnitude thereof is considered, in that it is the greatest circle, & hath these 2. proprieties, the one, that it diuideth the sphere into 2. equall partes, the other, that it hath the same centre with the world.

The situation thereof is in the midst betweene both the poles of the world, in which respect it differeth from the rest of the paralleles, and oblique circles.

The equall conuersion thereof, is that perfect reuolution which it fulfilleth within the determinate space of 24. howers.

The commodity thereof is great & manifold, and it is either Astronomicall, or Geographicall.

The vse therof in Astronomical matters, is seen chiefly in 4. thinges.

First, by the helpe thereof we vnderstand the measure of the first motion, and thereby recon the time, which is the measure of the first motiō.

Secondly, it helpeth vs in the finding out of the Equinoctialls, and that in two respects.

The one, in respect of the whole Earth. For euery Horizon of euery countrey diuideth the Equator onely of all the paralleles, into 2. equall pieces,

pieces, whereby it commeth to passe, that when the ☉ is in the Equinoctiall, the day & the night throughout the whole world are equall.

The other in respect of certaine Regions. For those that dwell vnder the Equator, in what part of the heauen soeuer the ☉ is, haue alwaies the *Aequinoctium*, or the day and the night equall.

Thirdly, thereby we find out both the situatiõ of the Stars either toward the North, or toward the South, because it distinguisheth the North part of the world from the South: as also their declination, either Septentrionall or Meridionall.

Fourthlye, throughe the helpe thereof, wee search the length of the artificiall day.

The vtility therof in Geographie is seene in 3. thinges.

1 Thereby we set euery town in his due place, in the terrestrial Globe.

2 It bringeth vs vnto the knowledge of all the paralleles, aswell celestiall as terrestriall.

3 By the ayd thereof we finish the description of the earth.

Of the greatest circle measuring the second motion. Chap. 18.

THe starres of heauen which are mooued round about from the West towarde the East, doe describe in the midst betweene their poles, a certen circulare surface common to all the planets, and a certen circulare line propre vnto the ☉ onely.

Con-

1 Concerning the circulare surface, there are deliuered by the Astronomers to be considered, the names, the definition, the measure, and the vse thereof.

The names are diuerse, drawen either from the Greekes, or from the Latines.

The Greekes call it the Zodiake, either of ζωη, lyfe, for that it is the path wherein the ☉, (taken to bee the author of life) doth walke: or else of ζῴδιον, a liuing creature, for that the ancient Astronomers, haue beautifyed this circle with the figures of certaine liuing creatures.

The Latines tearme it *Signifer*, as carrying the signes, they call it also the oblique circle, or the circle leaning a side.

The definition conteineth the magnitude, the oblique situation, and the limites thereof.

Concerning the magnitude therof, this is onely to be considered: that it is one of the greater circles.

The oblique situation thereof, is either in respect of the paralleles, which it cutteth at inequal angles: or of the irregulare ascensions, and descensions, or of the poles of the world, from the which it is not equidistant.

The limites thereof are the 2 tropicks, which it toucheth, and diuideth the Equator into two equall partes, declining therefrom by little & litle vnto the distance of 31. gr. 28. mi.

The measure thereof is either in regard of the length that it hath, or of the breadth.

The length thereof is 360. gr. and is diuided into 6. Northren signes, as ♈. ♉. ♊. ♋. ♌. ♍.

&

& into 6. Southren signes, as ♎. ♏. ♐. ♑. ♒. ♓.

The breadth thereof is 16. gr as wel in regard of the roming of the planets from the Ecliptick, and specially of ♀. & ♂. as also in respect of the principall constellations, whereof the greater part declineth from the midst of the zodiake.

The vse is chiefly seene in the obliquitie thereof: for thereby it falleth out, that the partes of heauen, do with the more ease maintaine their course against the first and vniuersall motion: as also that the starres may sometimes bee in the South, and somtimes in the North, for the greater benefite of the inhabitants of the earth.

The circular line propre vnto the ☉ onely, hath diuers names, with the definition & commodity peculiarly appertaining thereunto.

It is called the wheeling, the way, the course, the place of the ☉, the Eclipticke line, also the burnt line, and the diuision of the zodiak.

The definition thereof, is that whereby it is called a greater circle, diuiding the breadth of the zodiake into 2. equall partes.

The commoditie thereof is noted as well in designing the Eclipses of the ☉ & ☾, which neuer happen but when both of them are vnder or very neere the Ecliptick line; as also in distinguishing of the 4. quarters or seasons of the yeare.

Of certaine termes whereby the starres haue relation vnto the aforesaid circles. Chap. 19.

THE whole number aswel of the fixed starres, as also of the planets, hath relation both to the

the

the Equator, and to the Zodiak.

They haue a twofold relation vnto the Equator, either in regard of the orbiculare longitude of the Equator, or of the laterall position, therof.

In the orbiculare longitude of the Equator we are to note the names, and the definition.

It is sometimes called the longitude of a star: and sometimes the right ascension.

It is defined: The arke of the Equator comprehended betweene the head of ♈, and the section of a great circle passing through the poles of the world and the true place of the star.

The lateral position hath also name, definition, and diuision.

It is called the declination of a starre.

It is defined to be: The arke of a great circle, passing through the poles of the world, and the true place of the starre, the saide arke being intercepted betweene the Equator, and true place of the starre.

It is diuided into the Septentrionall, and Meridionall declination.

The relation that the starres haue vnto the Zodiake, is also two folde, either according to the Longitude of the Zodiake, or els according to the transuerse distance towarde either of the Poles.

In the Longitude of the Zodiake we are to consider the name, and the definition.

It is called Longitude: For that it is reconed longwaies on the circumference of the Eclipticke: it is also called the true motion of the Starre.

It

It is defined to be the Arke of the Zodiake, intercepted betweene the head of ♈, and the section of a great circle passing through the poles of the Zodiake, and the true place of the starre.

In the transuerse distance we are to note the name, the definition, and the diuision.

It is called Latitude, because it is reconed according to the position that it hath from some one side of the Ecliptick.

It is defined to be the arke of a great circle, drawen through the poles of the Zodiake, and the true place of the starre, the said Arke being intercepted betweene the Zodiake, and the centre of the Starre.

It is diuided into the Septentrional Latitude, when the starres are vnder the northerly signes: and into the Meridionall Latitude, when they are in the Southerly signes.

Of the proportion and supputation of the declination of euery point of the Eclipticke, or the regarde of the partes of the Zodiake, vnto the Equator. Chap. 20.

IN the declination of any point of the Eclipticke, 2. thinges are to be obserued: the proportion, and the supputation.

In the proportion we may note also 2. things: For either they haue none obliquation, or els their obliquations are equall.

Those that haue none obliquation, are the head of ♈ and ♎, as being the common intersections of the Equator and the Zodiake.

Those that haue equall obliquations, are such as are equally distant from the Equator, and they are either greater obliquations, or els the greatest.

The greater obliquations are those that haue any distance lesse then the greatest from either of the sections, and of that sorte there are alwaies foure.

The greatest obliquations are those that haue the greatest distance from the Equator, as the head of ♋, that is, the Somer solstice: and the head of ♑, that is the winter solstice.

The supputation is made either by the tables of declinations, or of Sines.

The Tables of declinations are calculated in sundrye places by Astronomers, and they consist of the 2. sides, the Area, and of the two extremities or endes.

The sides are either at the right hand, or at the left: that at the left hand, to be entred into, whẽ you haue the signe in the toppe of the table: and that on the right hand, when the signe is in the foote therof.

The Area is that, wherein at the common angle the declination is found.

The 2. extremities are those that conteine the signes: of which extremities, the one is called the toppe, or vpper parte: the other the foot, or the nether parte of the table.

The supputation that is made by the table of Sines, is performed by the helpe of the rule of 4. proportionall numbers, wherein 3. numbers are giuen, and fourth is to be sought out.

The 3. numbers giuen, must conteine the right sine of the whole quadrant, or of the semidiameter: the right sine of the greatest declination of the ☉: and the right sine of the distance of the point of the Eclipticke giuen, from the first section of the Zodiake and the Equator.

The fourth number produced by multiplication and diuision, is the right sine of the declination sought, whose subtended arke declareth the number of degrees.

Of the 2. circles called the colures, distinguishing the Equinoctiall and Solstitiall pointes. Chap. 21.

FOrasmuch as there are certaine pointes of the Zodiake and the Equator more notable then the rest, therfore the Astronomers haue thought good to fit vnto those pointes 2. Circles, wherof we may consider the reason of their name, their definition, their number, their figuration or description, and their vse.

They are termed colures, that is imperfect, in 3. respectes.

1. Because they appeare alwaies incomplete, or maymed, the which thing notwithstanding semeth to be common with diuers other circles.

2. Because they haue some partes that do neuer arise.

3. Because they are carried about after an imperfect manner, & not according to the position of Longitude, as the motion of the Heauen is.

The definition conteineth their magnitude, their

their intersection, and their motion.

As touching their magnitude, they are of the number of the greater circles.

As touching their intersection, they cut one another in both the poles of the world, at sphericall right angles.

In their motion, they are moued together with the sphere.

Their number is two: wherof the one passeth through the Equinoctiall pointes and the poles of the world, and is called either the equinoctial colure, or the distinguisher of the Equinoctialls: the other passeth through the solstitiall pointes, and the poles both of Eclipticke and of the worlde: and is called both the solstitiall colure, the distinguisher of the Solstices, and also the circle of the greatest declinations.

Their figuration is described by the semidiameter of the worlde, whose reuolution being fullye perfourmed through the poles of the worlde and the Equinoctiall pointes, maketh the Equinoctiall colure, but passing through the poles of the worlde, and the solstitial points, it maketh the solstitiall colure.

Their vse is manifolde, but principallye in 3. thinges.

1. In distinguishing the Equinoctiall and Solstitiall pointes.

2. In reconing aswell the quantitie of the greatest declinations of the ☉, by the arke intercepted betweene the Equator and the Eclipticke: as the space comprehended betweene the poles of the worlde, and the poles of the Eclipticke

tick, which is alwaies equall vnto the arke of the greatest declination.

3. For better vnderstanding of the ascensions and descensions of the signes.

Of the Meridian. Chap. 22.

THe ☉ carried about by the first motion, whẽ it is at the highest, designeth a point of a circle, whose definition, varietie, and office, we are to consider.

The definition taketh holde both of the names thereof, and of the matter it selfe.

It is called the circle Meridian, Meridionall, and Merinoctiall, the circle of the midday and midnight, either because it diuideth both the day and the night into 2. equall partes, the one ascending, the other descending: or els, because so often as the ☉, according to the first motion, is vnder the Meridian, it is then either midday, or els midnight.

The matter it selfe is that, according whereto it is defined to be one of the greater circles, drawen through the poles of the worlde, and the verticall point of any place geuen, and standing still when the Sphere is moued.

The varietie of the Meridian, by reason of the round figure of the earth, is either none at all, or manifolde.

It is none at all, either in regarde of reason, or of sense.

It is none at all in regarde of reason, when one place is distant from another in Latitude onely, that

that is, from the North to the South, or contrariwise.

It is none at all in sense, when one place is distant from another according vnto Longitude, which is from the East vnto the West, or contrariwise 36. scrup. that is, about 300. furlongs.

The varietie is manifolde in regarde also of reason and of sense.

The manifolde varietie in regarde of reason is, when examining the least distance towarde the East or West, we conclude another Meridian: and by this meanes we may haue so many meridians, as there shalbe places at euery small distance toward the East.

The manifolde varietie according vnto sense, is as often as any two places shalbe distant one from another, betweene East and West, more thẽ halfe a degree, and by this meanes we may haue so many meridians as there are halfe degrees of the Equinoctiall circle.

The office of the Meridiane is twofolde, either Astronomicall, or Geographicall.

The Astronomicall office thereof is executed two manner of waies.

1. In pointing out the Noones tide, or Midday, either naturall or artificiall.

2. The diuers habitudes and positions of the starres, following the motion of the heauen it selfe, are ascribed vnto the Meridian.

The Geographicall office therof is also of two sortes.

1. By the helpe thereof the Longitude of all places is calculated: and what places are more oriental,

orientall, and which more occidentall.

2. By the aide thereof we describe in the terrestriall plane, a correspondent merdiane line, for diuers vses of Astronomicalli Instrumentes.

Of the Horizon. Chap. 23.

There is also another circle, which the ☉ by the firste motion dooth point out in the East and West pointes, whose definition, diuision, and dignitie, is to be considered.

The definition stretcheth it selfe both to the names therof, and to the matter.

It hath diuers appellations: and is sometimes called the Horizon, *Finitor*, *Finiens*, as limiting our sight, and sometimes the compasse or circle of the Hemisphere of diuers regions.

The matter it selfe attendeth the description of the centre or pole thereof, the circumference and the magnitude.

The centre or pole of the Horizon, is the verticall point of eache place, distant from the Equator so much, as the poles of the world are distant from the Horizon.

The circumference of the Horizon is that, which the semidiameter of the worlde in his full reuolution through the pointes of the East and West, and the rest of the brymme of Heauen, describeth.

The magnitude of the Horizon is considered, in that it is one of the greater circles, diuiding the worlde (in regarde of sense) into 2. equall segmentes, wherof the one is called the vpper, the

the seene, or the diurnall segment, the other, the lower, the hidden, or the nocturnall segment.

The diuision of the Horizon is considered, in respect of the Equator as it is either a right, or an oblique Horizon.

The right, or orthogonall Horrizon hath 3. proprieties.

1 With the Equator it hath equal angles.

2 It hath the pole, or the verticall point in the Equator.

3 It hath the poles of the world in his circumference.

The oblique, bending, or inclining Horizon, is in all thinges contrary vnto the right.

The dignitie of the Horizon, by reason of the manifold vse thereof, is great: For by the helpe thereof, we learne 6. thinges.

1 The quantityes of the artificiall daye and night, and consequently the time of the rising, and setting of the ☉.

2 The equall hower of the day, the ☉ shining

3 The degree of the Zodiak, wherewith anie starre geuen doth arise and sette.

4 What starres do alwayes appeare, or are alwaies hidden.

5 The rising and setting of the starres.

6 The Eclipses of the ☉ & ☾, either seene, or not seene.

The diuision of the Horizon, according to Proclus. Chap. 24.

Moreouer, the Greekes deliuer vnto vs a more subtile diuision of the Horizon: and it is twofold, the one to bee conceaued in in mind, only, the other falling within the compasse of our sense, or our sight.

Concerning the Horizon to be conceaued in minde onely, wee are to note the names, the description, the cause.

It is diuersly named, as either rational, or conceaued by reason, by the Greekes called Ιηισθημα-νικὸς, and also naturall.

The description thereof is absolued by a semidiameter, and a circumference and the Area thereof.

The semidiameter is that line, whereof the one extremity is in the eies of the inhabitants of the world, the other extremity is in the orbe of the fixed starres.

The circumference and the Area is the space and compasse, which the semidiameter maketh, beinge carryed about by the brimme of that part of the heauen, that is extāt aboue the Horizon.

The cause alleaged is, that our sight beinge inhable to pearce vnto the beholding of all the fixed starres, doth concludethat there is a certen circle in the heauen, that limiteth the thinges seene, from the thinges not seene.

In the Horrizon apprehended by our sense, we are to note the names, the description, the varietie.

It is called the Horizon sensible, or perceiued by our sense, also the apparent, and artificiall Horizon.

The description is perfourmed by a semidiameter

meter, a circumference, and a plane.

The semidiameter is that line, whereof the one limite is in our eye, the other is in the end of our sight vpon the surface of the earth, consisting of a thousand furlongs, which ende wee imagine in a free prospect, to ioyne the heauen and the earth together.

The circumference and plane is that space & compasse, which the aforesaid semidiameter turning about, doth describe.

The variety is common aswel vnto the Rationall, as the sensible Horizon, & it is either none at all, or else manifold.

The variety of the sensible Horizon is said to bee none at all, when the Horizon doth continue all one and the same, and it is ether in reason, or in sense.

The sensible Horizon is not varied in reason, when the places are not any whit, nor any way changed.

The sensible Horizon is said not to be varied in sense, when the places distant about 400. furlonges one from another (that is 48. mi.) do not alter, either the climate, or the length of the dayes, or the apparences of the heauens.

The variety of the sensible Horizon is manifold, when rhe places are varied more then 400. furlonges, and are situated either towarde the East, or West: in which variety neither the climate, nor the length of the day, nor the apparences of the heauens are changed with the Horizon: or else they are situated toward the north or south, wherin together with the Horizō both

both the climate, and the length of the dayes,& the apparences of the Heauens are altered.

Of the two Tropickes. Chap. 25.

THe ☉ carryed about by the second motion, in his greatest declination from the Equator by the violence of the first motion, describeth certaine paralleles, whereof the generall reason, the number, & the offices are to be considered.

The generall reason is ether in respect of their names, or their definition.

They are named by the Greekes τροπικοὶ, Tropickes, by the Latines *Versiles, conuersiui, Vertentes*, tourning, and the Solstitiall paralleles.

Their definition conteineth their quantitie and their circumference.

Their quãtity is noted, either in respect of the other circles, these being compted in the number of the lesse circles, or in regard of thẽselues, whereby they are compted equall, in asmuch as they are equally distant from the centre of the world, beeing separated the one from the other by the double distance of the ☉ greatest declination.

Their circumference is that round compasse, which the ☉, passing throughe the 2 solstitiall points, doth describe.

They are in number 2. the one Septentrionall, the other Meridionall.

The Septentrionall Tropicke is on this side of the Equator (in respect of vs) which wee call either the Sommer tropicke, for that it passeth through

through the poinct of the Sommer solstice, or els the tropick of ♋, because it is described through the end of ♊, & the beginning of ♋.

The Meridionall tropicke is situated on the other side of the Equator, and is called either the Winter tropicke, as passing through the poinct of the Winter solstice, or the tropicke of ♑, because it is drawen through the head of ♑.

The offices and commodities of them, are in number 4.

1 They shewe the Tropes, that is, the conuersions, or tournings of the ☉, aswel in Sommer, happening in our age the 3. and 2. of the Ides of Iune, as also in winter, the 3. & 2. of the Ides of December.

2 They shew in euery situation of the sphere, both the longest day, which is as long as the diurnall Arke of the Tropicke of ♋, conteineth howers: and the shortest day, which is as long as the space of howers, conteined within the diurnall arke of the tropicke of ♑.

3 They poinct out the limits of the course of the ☉, and his greatest declinations: which are 23. gr. 52. mi. as in the time of *Aristarchus & Ptolemee*, or 23. gr. 28. mi. as it is now in our time.

4 They shew the burnt zone which they separate from the temperate, and the midst of the second climate, which they call *dia-Syenes*, and *Anti-dia-Syenes*.

Of the 2. polare circles. Chap. 26.

THe two poles of the Zodiake, carried about by the regulare reuolution of the vniuersall frame

frame, describe about the poles of the worlde, two circles, whereof the generall reason, the number, and the vse is to be noted.

The generall reason offereth to our consideration, their name, their definition, and their accidents.

They are called the Polare circles, either because they are described about the poles, or by the poles.

Their definition by the Latines, is made by their quantitie, and their circumference and plane.

Touching their quantity, they are in the number of the lesse circles, equall in all places.

Their circumference and plane is described either by lines, drawen from the poles of the Zodiake, vnto the Axe of the worlde, at right angles, and hauing by the daily motion a perfect reuolution: or els they are described by certen semidiameters, drawen from the centre of the earth vnto the poles of the Zodiak, and turned about by the diurnall and nocturnall motion.

The accidentes of the polare circles do determine either their equality, for they are paralleles, compared either one with another, in asmuch as they are equidistant from the centre, or compared with the tropicks, & the Equator: or else they determine their distance, either from the next tropicke, which is 43. gr. or from the poles of the worlde, which is equall vnto the ☉ greatest declination.

They are two in number: The one Septentrionall,

nall, the other Meridionall.

The Septentrionall Polare circle is described by the North pole of the Eclipticke: the Meridionall, by the South pole thereof.

The Septentrionall polare circle is called Boreall, North of the North winde called *Boreas*, and Arcticke, and Septentrionall, because of the 2. constellations, the one of the greater beare called *Arctos*, the other of the lesse beare called *Septentriones*, which are nigh thereunto.

The Meridional polare circle is called *Australl*, or Southerne of the South winde called *Auster*, and Antarcticke, is opposite vnto the Arcticke, and Meridionall, of the South part of heauen, called *Meridies*.

Their vse is noted in that they comprehende the cold and frozen zones, and the inhabitants of the earth called *Periscii*, whose shadowes goe round about them, and on either side limite the distances of the poles.

The Polare circles otherwise described according to the Grecians. Chap. 27.

THe polare circles are described, either according to the greatest declination of the ☉ or the altitude of the Pole aboue the Horizon.

The greatest declination of the ☉, by meanes of the motion of inclination of the eight Sphere, is diuerse. For it was one in times past, and is found to be another now: and of such circles, the reason is declared in the Chapter before.

The polare circles described according to the altitude of the pole, require the consideration of of their definition, their varietie, their offices, and the manner of their description.

In defining the Arᶜticke polare circle, we say: 1. That it is the greatest of those circles which are alwaies in our sight, that is, of those which we may see at the same instant: 2. that it toucheth the Horizon in one point: 3. that it is altogether aboue the earth.

In defining the Antarctick polare circle, we say: 1. that it is equall and parallele vnto the Arcticke: 2. that it toucheth the Horizon in one point: 3. that it is altogether hidden vnder the earth.

The varietie is manifolde, according to the didiuersitie of the climates. For either they are not at all, as in a right Sphere it happeneth, where excluding altogether the polare circles, the Grekes recon 3. paralleles onlie: or els they are, and those sometimes either lesse, equall, or greater then the tropickes, or els they are equal vnto the Equator and the Horizon. For by how much the pole shalbe higher, by so much shal these circles be greater.

The offices and vse of the Arcticke circle is, in that it sheweth the Starres that neuer arise nor sette: of the Antarcticke circle, the contrary is to be conceiued.

The meanes of their description is by those Starres, that in any Region do touch the Horizon.

Of the Milkie circle. Chap. 28.

OF all the circles, there is none to be seene, beside the Milkie circle, which for that the Greekes

Greekes do recon among the other circles, we wil expresse the names, the definition, the causes thereof, and the distinct Starres which make the same.

The names are diuerse: as *Galaxia*, the Milkie orbe or circle, the Milkie Zone or milkie waye. The Arabians call it *Mataratì*, as it were a broad, space, or arke that moueth.

It is defined to be one of the greater circles, oblique, drawen or stretched toward both the Poles, most brightly shining, apparent vnto the sense, inequall, both in breadth and in colour.

The causes are diuers, and those either fabulous, or naturall.

The fabulous causes are in number 4.

The first is taken from the scorching of the ☉, as if the ☉ had sometimes made his motion there, and by his scorching had caused that place to be white.

The second is drawen from the milke of *Iuno*, that running plentifully out of her pappes, painted this circle of that colour.

The third is fetched from the seate and habitation of strong and valiant men, whom the Poets haue placed in this circle.

The fourth is deriued out of the way of the Gods, as if they passed thereby vnto the pallace of *Iupiter*.

The naturall causes alleadged (although they be many, yet) are principally, but 3.

The first by *Theophrastus*: who said, that it is that ioyning together, wherby the heauen being diuided into two hemispheres, is as it were by a certen

certen claye fastened.

The second, by *Aristotle*: who tooke it to be a Meteore, set on fire in such sorte as a Comete,

The third is Astronomicall: which affirmeth that it is a girdle caused by many little starres, as it were one touching another, in the which concurring in that Place, the light of the Sunne is diffused.

The distinct starres that make it, are cheiflie these: The *Arowe*: the *Eagle*: the bowe of ♐: the *Altare*: the 4. feete of the *Centaure*: the ship *Argo*: the head of the *Dogge*: the right hand of *Orion*: *Erichthonius* or the *Wagoner*, with the *Goate* on his shoulder: *Perseus*: *Cassiopeia*: and the *Swanne*.

Of the 5 principall Regions of the worlde, commonly called Zones. Chap. 29.

THe Vniuersall Globe aswell of the heauens, as of the earth answerable thereunto, is distinguished into certain orbicular tractes, which the spaces comprehended betweene the 4. paralleles do make, of which tractes we may consider the names, the definition, the generall nūber, and their distance one from another.

Their names are diuerse: For they are called either Zones, or swadlingbandes, or girdles, or *Mashes*, or coastes.

They are defined to be the space either of the heauen, or of the earth, comprehēded betweene two lesse paralleles, or els included on euerye side with the polare circles.

Their

Their generall number is twofolde: For either they are celestiall, and so the causes of the terrestriall, or els they are terrestriall, of the same proportion with the celestiall.

The celestiall are either Meane, or Extreme, or betweene meane and extreme.

The Meane is that Zone which is included betweene the 2. tropickes, and is cut in two equall partes by the Equator,

The Extremes or polare Zones, are those wherof (being but 2) the one is called the Septentrionall Zone, within the Arcticke circle: the other the Meridionall Zone, within the Antarcticke circle.

The Zones between meane and extreme, are also 2. whereof the one is Septentrionall, comprehended betweene the tropicke of ♋, and the circle Arcticke, and the other Meridionall comprehended betwene the tropicke of ♑, and the circle Antarticke.

The terrestriall Zones haue the same reason with the celestiall, aswell in respect of their nūber, as in regarde of their names.

The terrestriall Zones are also 5. in number, answering proportionallye vnto thé 5. celestiall Zones, conically marked out by the 4. celestiall paralleles.

The terrestriall Zones haue the same reason with the celestiall, in respect of their names also: For that terrestriall Zone that is vnder the mean celestiall, is called meane: those which are vnder the extremes or polares, are called extremes septentrionall, or Meridionall: and those which

are

are vnder the Zones betweene meane and extreme, haue their name accordingly, and are either Northerlie, or Southerlie.

The distance one from another is in this manner: the meane or burnt Zone, according to the Latitude reconed in the Meridian, conteineth 47.gr. or 705. miles: the extreme intemperate Zones do each of them, according to the said reconing conteine, as many degrees and miles, as the meane: the temperate zones betweene meane and extreme, do eache of them conteine according to the former reconing 41. gr, or 645. miles.

The difference of the Zones, and the manner how all places vpon the earth, may be brought within their compasse. Chap.30.

THe difference also of the zones aswell celestiall as terrestriall, and the reason how all places vpon earth may be referred vnto them, is worthie the noting.

Their difference is to be considered either in respect of their figure, or their accidental nature.

The figure of the meane is vniforme, and for the most parte alike.

The figures of the extremes are either of thē equall to other, yet such, as that they seeme rather to carie the shape of circles, then of zones.

The figures of the zones betweene meane and extreme, be either of them alike, and equall vnto the other: yet about the tropicks their figure is limited with a greater compasse, then towarde the

the polare circles.

The accidentall nature of the zones is that, in regarde whereof they are saide to be meane, extreme, and betweene meane and extreme.

The Meane or burnt zone is diuided into 2. partes, whereof the one is situated vnder the Equator, the other about the Tropickes.

That parte which is situated vnder the Equator seemeth to be temperate, and that for three causes.

1. By reason of the sodaine and crosse accesse, and recesse of the Sunne.

2. By reason of the continuall equality of the night and day in that place.

3. By reason of the swift carying about of the ☉, by the first motion.

That parte which is situated vnder the Tropickes is hardlye to be inhabited, and that also for 3. causes.

1. For the slowe conuersion of the ☉.

2. For the doubled proiection of the Sunnebeames, vpon those places.

3. For the great increase of the Sommer daies aboue the nights.

The extreme zones are both of them frozen, by reason of the too much colde that falleth out there, by meanes of the oblique proiection, and reflexion of the Sunne beames.

The zones betweene meane and extreme are both of them temperate, and are diuided into 3. Regions, whereof one is situated about the middle parte thereof, which we iudge simplie to be temperate, by reason of the moderate heate of of the ☉, namely, from 34. gr. vnto 48. gr. distance from the Equator: the other 2. regions are about the extremes therof, the one being about the tropicks, and so subiect vnto the intemperate heate of the burnt zone, the other nigh vnto the polares, and therfore subiect vnto the intemperate colde of the frozen zone.

The reason how al places vpon the earth may be referred vnto those zones, hath two considerations.

1. If the places haue Septentrionall Latitude, and that lesse then the greatest declination of the ☉, they belong vnto the burnt zone: if equall, vnto the trop. of ♋: if greater, and yet not exceeding 6 ., gr. 30. mi. they belong vnto the temperate zone. If the said septentrionall Latitude be equall vnto the complemēt of the greatest obliquation, they must be placed vnder the arctick circle: if greater, vnder the frozen zone.

2. If the places giuen haue Meridionall Latitude, the same Iudgement is to be pronounced of them, as of the places vnder Septentrionall Latitude.

Of the fowerfolde rising and setting of the Starres. Chap. 31.

THe Poets, and for the better parte all other Authors, doe periphrasticallye describe the times of thinges, worthie the noting, by the Starres of heauen, either rising or setting.

In their rising is to be considered, the definition, the subdiuision.

The definition doth cheiflie consist of the name, and of the matter.

The name in this place signifieth their first apparition vnto the eie, or their Ascension.

The matter is that according wherunto, the rising of a starre is defined to be the apparition of any starre giuen, which before could not be be seene, as either being vnder the Horizon, or hidden by the Sunne beames.

The subdiuision also offreth 2. considerations.

1. That the starres do ascend or rise by the vniuersall motion from the lower hemisphere vnto the Horizon, either in the morning with the ☉, and then they are said to haue a morning, a diurnall, a cosmicall, or worldly rising: or els in the Euening at the ☉ setting, and then they are saide to haue an euening, a nocturnal, a chronicall, or acronychall rising.

2. That the starres do rise by the 2. motion freed from the ☉ beames, either before the rising of the Sunne, and then they are saide to haue an Heliacall morning rising, which commeth to passe in those starres that are slower then the ☉, or els after the setting of the ☉, and then they are saide to haue an Heliacall euening rising, and that is in those starres, that are swifter thē the ☉.

In the setting of the starres there is also offred the definition, and the subdiuision.

The setting is defined to be: the occultation or hiding of any starre giuen, either by the depression therof vnder the Horizon, or by the ingression thereof into the beames of the ☉.

The subdiuision consisteth in their setting and withdrawing from our sight, which is done two maner manner of waies.

1. By the firſt motion they deſcende from the vpper hemiſphere vnto the lower, either in the morning which is done coſmically at the riſing of the Sunne, and that ſetting, as the riſing alſo, is referred vnto the ☉, and thoſe ſignes of the Zodiake which the ☉ poſſeſſeth: or els in the euening, which is done chronically, at the ſetting of the ☉, and this ſetting, as alſo the riſing, is referred vnto all the ſtarres generally.

2. By their propre motion at their entrance into the beames of the Sunne, either before the ſunne riſing, that is, coſmicallye, which happeneth onely vnto the ſtarres that are ſwifter then the ☉, or els after the ſetting of the Sunne, that is, chronically, which belongeth vnto thoſe ſtarres onlie, thet are ſlower then the ☉.

Another more eaſie and perfect diſtinction of the riſinges and ſettinges, with the expoſition of certen principles which are to be vnderſtoode for the reading of Authors, concerning the riſing and ſetting of the Starres, taken out of Ptolemee, *and the later Aſtronomers.* Chap. 32.

FOr the eaſier vnderſtanding of the Poets and other Authors, which by the riſing and ſetting of the ſtarres do circumſcribe the times, 4. things chieflie are to be knowne.

1. The latitude of the place wherof the ſpeach is made, which may be gathered out of the Tables of the Regions, ſet downe in all Geographicall writings.

2. The place of the ☉ in the Eclipticke at any time, which the ancient Recordes do miniſter, where notwithſtanding you muſt note, that our age doth differ from former times: and that the ☉ in our age doth entre into the heades of the ſignes, ſooner almoſt by 6. daies, then in the ancient times.

3. What ſignes are oppoſite one vnto another: viz. ♈ to ♎: ♉ to ♏: ♊ to ♐: ♋ to ♑: ♌ to ♒: & ♍ to ♓.

4 The difference of the riſing, or of the ſetting.

The riſing is either Heliacall and of the Morning, or Acronychall and of the euening.

The Heliacall or morning riſing, is either true or apparent.

The true Heliacall riſing is when a ſtarre ioyned with the Sunne, doth together and at the ſame inſtant ariſe with him in the morning.

The apparent Heliacall riſing, is when the ſtar doth aſcende and begin to appeare at the dawning, and before the Sunne riſing.

The Acronychall or euening riſing, is alſo either true, or apparent.

The true Acronychall riſing, is when a ſtarre preciſely riſeth, at the very inſtant of the Sunne ſetting.

The apparent Acronychall riſing is when after the ſetting of the Sunne, the ſtarre being freed from the beames thereof, ſhall make his firſt apparence in the twilight.

The ſetting of a ſtarre is alſo either Heliacall, or Acronychall.

The Heliacal ſetting is either true or apparẽt.

The true Heliacall setting is whē a star at the ☉ rising, doth at the same instant set in the opposite part of the world, which before was called the morning starre.

The apparent Heliacall setting is when in the morning, somewhat before the ☉ rising, the starre is newly seene to set.

The Acronychall setting is in like sorte either true or apparent.

The true Acronychall setting is when at the ☉ setting, the starre also setteth, which all the meane time was called the euening starre.

The apparent acronychall setting is when after the setting of the ☉, the starre doth not set at the same instant with the ☉, but by reason it is hidden by the beames of the ☉, it appeareth no more vntill the morning that it arise againe.

Of the Astronomicall rising & setting of the signes: or as the Greekes call it, ἀνωίαι καὶ πραγματικὰ τῶν διαφορῶν καὶ καταφορῶν. Chap. 33.

THe rising, the comming vnto the Meridian, and the setting of the signes, or of any point of the heauē, is either poetical, or astronomicall.

The Poeticall or vulgare, is when the reason of the apparition, or occultation of the signes, is onely in their comparison with the ☉, which was handled in the 31. & 32. chapters.

The Astronomicall rising, culmination, & setting of any starre or point of the heauen is that which defineth the proportion of the time and space, both when and how great it is, wherein the

the aforesaid thinges are performed, either in a right, or an oblique sphere.

In the rising are to be considered the definition, and the bipartite diuision.

The definition is either of the name, or of the matter.

The ascension is called the rising, which wee measure by the coascendēt arke of the Equator.

The matter is that, according whereunto it is defined, to be the arke of the Equator, comprehended between the signe rising, or the East part of the Horizon that conteineth the signe, & the head of ♈, the which arke is to be accōpted according to the orderly succession of the signes.

The consideration had of the diuision, is that either a greater portion of the Equator riseth with the signe, & then it is said to haue a right ascension, because it maketh righter angles with the Horizon: or els that a lesse portion of the Equator doth ascende therewith, and then it is said to haue an oblique ascension, by reason of the more oblique angles that it maketh with the Horizon.

The culmination is defined, either the passing of some poinct of the Zodiake, or of the world by the Meridian circle, or else the degrees of the Equator, which with the portion of the Zodiake geuen, passe through the Meridian.

The setting of a signe or of any poinct of the heauen, offereth 2. thinges vnto our consideration, the definition, and the diuersity thereof.

The definitiō is either according to the name, or according to the matter.

 Accor-

According to to the name, it is called the descension or setting, which wee measure by the arke of the Equator descending therewith.

According to the matter it is defined to bee the arke of the Equator, cōprehended between the signe or poinct setting, and the head of ♈.

The consideration of the diuersity of settinge, is either that a greater part of the Equator descendeth with the signe or poinct of the heauen, and then it is said to haue a right, or a long, and slowe descension: or els that a lesse portion of the Equator setteth therewith, and then it is said to haue an oblique, or a short and swift descension.

Of the diuersitie of ascensions, descensions, and culminations, in a right sphere. Chap. 34.

THe Zodiake in a right sphere is fitted vnto the equall conuersion of the Equator, and together with the partes thereof, passeth by the East, or the West, or the midst of heauen, both in the quadrants or quarters, and in the signes.

The quadrants in equall spaces of time do ascend and descend, or do passe through the midst of heauen, beginninge either at the Solstitiall poinctes, namely at the heade of ♋ & ♑, and compting to the end of ♍ & ♓: or els, beginning at the Equinoctiall poincts, which are the heads of ♈, & ♎, and compting to the ende of ♊ & ♐.

The signes applied vnto the motion of the Equinoctiall, are considered either whole, or in partes.

The signes considered wholly, haue relation either

either vnto the Equator, or vnto the Zodiake.

The signes in their relation vnto the Equator, do ascend inequally: For some of them doe rise rightly, and some obliquely.

Those that haue right ascension are ♊. ♋. ♐. ♑. with the which there do coascende 32. gr. 11.mi. of the Equator.

Those that haue oblique ascension, are ♈. ♎. ♓. ♍. wherewith there doe coascende 27.gr. 54.mi. of the Equator: and ♉. ♏. ♌. ♒. wherewith there arise 29.gr. 54.mi. thereof.

The signes in their relation vnto the Zodiake, or considered seuerally apart, haue ascensions, either equall or inequall one vnto another.

They haue equall ascensions, that come forth in equall times, and they are either opposite in the diameter, or equally distant from the Equinoctiall poinctes, as are ♓ ♈: ♒ ♉: ♑ ♊: ♐ ♋: ♏ ♌: ♎ ♍.

They haue inequall ascensions that neither are opposite, nor equally distant from the aforesaid poinctes.

The signes considered in their parts, haue also relation either vnto the Equator, or vnto the Zodiake.

The partes hauing relation vnto the Equator do (as before) ascend inequally, and that either rightly or obliquely.

The partes hauing right ascensions, are comprehended within the foure signes, nexte vnto the 2. solstitialles.

The partes hauing oblique ascensions, are conteined within the signes next vnto the Equinoctiall poinctes on each side.

Those partes of the signes, that haue relation vnto the zodiake, haue their ascensions partly equall, and partly inequall.

Partes hauing equall ascensions are these: the first degree is equall vnto the first degree of the opposite signe: and the first degree vnto the last of another signe equidistant from the equinoctial poinctes.

Partes hauing inequall ascensions, are those, in whom neither opposition falleth out, nor equidistancie.

Of the diuersity of ascensions, and descensions in an oblique sphere. Chap. 35.

IN the oblique situation of the sphere we consider either the proportion of the ascensions, or of the descensions of the zodiake.

The ascensions are compared and applyed either vnto the Equator, or one with another, or vnto the ascensions of a right sphere.

Beeing compared vnto the Equator, they are either equall, or inequall vnto the ascensions thereof.

In their equality they are numbred either in the Northren semicircle from the head of ♈, vnto the end of ♍: or from the head of ♎, vnto the end of ♓.

In their inequality, they are reconed either in the whole semicircles, beginning not in the Equinoctiall poinctes, but els where: or els the reconing is made, in some of their partes.

In their comparison one with another, they are either equall, or inequall.

When they are equall, they are reconed in some 2. concordant arkes of the Ecliptick, as in ♈♓: 14.gr.50.mi. ♉♒: 18.gr.51.mi. ♊♑; 27 gr. 16. mi. ♋♐: 36.gr.58. mi. ♌♏: 40. gr. 57.mi. ♍♎: 40.gr. 58. mi. in the latitude of 40.gr.

When they are inequall, they are reconed either in parts not equidistant, or in the semicircle either ascendent, or descendent.

The semicircle ascendent is from the head of ♑ vnto the end of ♊, and that ascendeth more oblique and swift.

The descendent semicircle is from the head of ♋, vnto the end of ♐, & it ascendeth more right and slow.

When the ascensions are compared vnto the ascensions in a right sphere, they are either lesse, or more oblique: or greater, or righter then the said ascensions in a right sphere.

The lesse or more oblique fall out in the North semicircle: the greater or more right happeneth in the South semicircle: the distance betweene the ascensions of each sphere, is called the difference of ascensions.

The descensions of the Zodiake, are vnto the ascensions thereof either equal, or inequall.

They are equall either in regard of the moities of the Ecliptick comprehended betweene the equinoctiall poinctes, or else according to the equidistant, or opposite partes of the Zodiake.

The descensions of the Zodiake are inequall, being compared either vnto the right sphere, or vnto the same climate.

The descensions of an oblique sphere are more oblique, then the descensions of a right sphere whereunto they are compared, when as the ascensions in an oblique sphere, are more right then in a right sphere.

The descensions of an oblique sphere are more right then the descensions of a right sphere, when as the ascensions in an oblique sphere, are more oblique then in a right sphere.

The inequall descensions of the zodiak, compared vnto the same climate are to bee noted, either in the parts of the Zodiake which descending oblique doe rise right, such as are the parts of the descending semicircle: or els in the parts of the zodiake, which descending right do rise oblique, and such are the partes of the ascending semicircle.

Of the naturall day, and of the inequality and difference thereof. Chap. 36.

OVt of the premisses wee may, not vnfitly, deriue some matter concerninge the dayes, whereof there are two sortes, the one is called ciuile or naturall, the other artificiall.

In the ciuile or naturall day, we may consider the definition, the distinction, and the cause of inequality.

The definition respecteth either the name, or the thing it selfe.

It is called either naturall, as caused by the naturall, or regulare motion of the whole: or νυχθήμερον by *Ptolemee*, as consisting of the night and day together: or els ciuile, because all nations naturally do tearm it a day.

The definition respecting the thing is that, according to which it is defined to be the space of 24. howers and certen minutes, consisting of light and darkenesse.

The definition thereof is in respect of the continuance and length of the day, and thereof one is called inequall, or different, also the true and apparent day (the Greekes call it ἀνώμαλον, irregulare:) another the equall, or meane day.

The inequall or different daye, is the space of 24. howers and so many minutes, as are answerable vnto each portion of the zodiak, which the ☉ doth daily run ouer.

The equall or indifferent day, is the space of 24. howers and so many minutes, as are answerable vnto the quãtity of the meane motiõ of the ☉ in one day, which is 59. gr. 8. mi.

The cause of the inequality happeneth vnto the true naturall day, either in a right, or in an oblique sphere.

The cause of the inequality happening in a right sphere, is through the inequall augmentation, by meanes either of the Equinoctiall ascensions inequally answering the same, by reason of the obliquitie of the zodiak: or els of the motiõ of the ☉, which about the cẽtre of the world is inequal

The cause of the inqualitye of the day happening in an oblique sphere, is through the inequall

quall augmentation apperteining either to the Equinoctiall ascensions inequally answeringe the same, by reason of the obliquity aswell of the Horizon, as of the zodiake: or else, to the eccentricke circle of the ☉ wherin the ☉ running doth in equall tymes, perform an inequall motion.

The Artificiall day is handled in the Chap. folowing.

Of the artificiall day and night, and the diuersitie belonging to them both. Chap. 37.

THe ☉ caried about by the first motion, distinguisheth the naturaall day into two partes, whereof the one is called the artificiall day, the other the artificiall night.

Concerning the artificial day, Astronomy deliuereth the definition, and the proportion thereof.

The definition conteineth the Author, and the terme thereof.

The Author of the artificiall day is the ☉, who caried about by the first motion, describeth in the day time a certen arke.

The terme is either from whence: that is, from the Easterlie part of the Horizon: or by what: that is, by the verticall meridian: or vnto what: that is, vnto the Westerly part of the Horizon.

The proportion of the artificiall day is deliuered in so much as apperteineth vnto the length thereof, either in a righte, or in an oblique sphere.

In a right sphere it is alwaies equall vnto it selfe,

selfe, and to the night, by reason of the equalitie both of the Ascensions (for the one halfe of the Equator doth alwaies equally ascend, and descend with sixe signes of the zodiake) and of the diurnall, and nocturnall segments.

In an oblique sphere the dayes to themselues and to the nights are either equall, or inequall.

The dayes are equall both to themselues and to the nightes in the Equinoctiall, by reason of the equality both of the ascensions (for looke how great the ascension of the diurnall arke is, so great also is the descensiō of the nocturnal) & of the segments which the ☉ describeth, the said segments being incident with the Equator.

The daies are inequal both among thēselues, and to the nightes, when the ☉ hath passed the Equinoctiall poincts, aswell by reason of the diuersity of the ascensions of the signes, as also by reason of the Sunnes inequall describing of the paralleles by the motion of the world.

The artificiall night geueth vs to consider the definition and the measure.

It is defined to be the part remaining of the naturall day, comprehending the space between the setting of the ☉, and the rising thereof.

The measure thereof is either equall, or inequall.

The equality of measure falleth out in the right sphere alwaies, in an oblique sphere two times in the yeare.

The inequality of measure hath notwithstāding either a like diuersity in the signes equidistant frō the Equator: or alternate in opposite points.

 Of

Of the reason of the equall and inequall howers.
Chap. 38.

HAuing thus set downe the description of the dayes, it falleth out nowe, to intreat of their partes commonly called howers, whereof we must consider the generall reason, and the diuision.

The generall reason attendeth their definition, their number, and their subdiuision.

They are defined to be that space of time, wherin the 24. parte, or 15. gr. either of the Equator, or of the Eclipticke, do fullie arise.

They are in number 24, belonging vnto euery naturall daye.

Euery hower is subdiuided into 60. minutes, euery minute into 60. seconds, &c.

The diuision of the howers consisteth in this, that either they are reconed in the Eclipticke, or els in the Equator.

Those that are taken in the Eclipticke, the ascensions whereof do varie, are called inequall howers, whereof the names, the definition, and the number are to be noted.

They are called naturall (by *Io. de sacro bosco*) and temporall, and artificiall, and Planetarie.

They are defined to be the space of time wherin the moitie of a signe of the Zodiake, counted from the place of the ☉, or the opposite thereof, doth ascende.

Their number is as much by day, as by night: For 6. signes of the Ecl. do alwaies arise, aswell by

by day, as by night.

The howers that are reconed in the Equator, which ariseth vniformelie, are called equall howers, whereof we are in like manner to note the names, the definition, and the number.

They are called naturall (by many) and equinoctiall howers.

They are defined to be that space of time, when in 15, gr. of the Equator do fully arise.

Their number is alwaies inequall, sauing in the 2. Equinoctiall seasons. For at other times, 6. signes of the Equator do not euerye daye completely arise and set.

Of the diuers accidents of diuers partes of the earth, according to the diuerse situation of the Sphere.
Chap. 39.

THe situation of euery place and region on the earth, is in the space either of the burnt, or temperate, or frozen zone.

The places situated in the burnt zone, are either in the meane spaces, or betweene meane & extreme, or in the extremes.

Their situation that are in the meane spaces, differeth from the rest: 1. In the 4. sortes of shadowes which they haue, viz. Septentrionall, Meridionall, Orientall and Occidentall: 2. In their 4. solstices which they haue, two being highest in ♈ & ♎, and two lowest, in ♋ & ♑: 3. In their continuall Equinoctialles: 4. In their two Winters, and two Somers.

Those that haue their situation betweene the meanes

meanes and extremes, do (as the former) differ from the rest: 1. In the double passage of the ☉ ouer their heads, but not in the heads of ♈ & ♎: 2. In their foure shadowes and Solstices, although not happening at the same time, as in former situation,

Those that haue their situation in the extremes of the burning zone, do differ from the other: 1. In that the Sunne commeth but once vnto their Zenith: 2. in the length of their greatest day, which is 3. ho. ½.

Those places that are situated within the temperate Zone, are either in the extremes, or in the meane.

The extreme spaces are those, that are vnder either the trop. of ♋, (wherof we spake before) or the Articke circle.

Those that are vnder the Articke circle do differ frō other: 1. In that they haue the Zodiake coincident with their Horizon, and the pole therof with their Zenith: 2. In that the signes do arise vnto them either most swiftlye, or most slowelye: 3. In the length of one daÿ, consisting of 24 howers.

Those that are situated within the meane spaces of the temperate zone, do differ from others: 1. In their verticall point, which the Sunne neuer cometh vnto: 2. In their shadowes, which are onlie 3.

Those places that are situated within the frozen Zone, are either in the meane spaces, or in the extremes.

Those that are within the meane spaces of the frozen zone, do differ from other: 1. In the intersection of the Zodiake, and the Horizon in equidistant pointes: 2. In that some portion of the Zodiake is alwaies either aboue the horizon, or vnder the same.

Those that are within the extremes of the frozen zone, are either vnder the Articke circle, (whereof we speake a little before) or vnder the pole.

Those that are vnder the pole do differ from other:

1. In their Horizon, which is all one with the Equinoctiall: 2. in their daye, which is halfe a yeare, by reason that the one moitie of the zodiake doth alwaies appeare aboue the horizon.

Of the diuersitie of the names of the inhabitantes.

Chap. 40.

THe inhabitants of the earth compared one with another, haue diuerse appellations, by reason aswell of the shadowes of the ☉, as of the Horizon, or paralleles and meridians.

The shadowes cast by the ☉ vpon the earth at Noone, are either infinite, or none at all, or els they are finite.

The shadowes that are infinite or equidistant vnto the beame, are cast in the frozen zones, whose inhabitants are called *Perisci*, that is shadowed round about, because their shadowes do goe in compasse round about them.

Those that haue no shadowes at Noone tide, are in the burnt zone, whose inhabitants are named either *Ascii*, because when the ☉ is in their

their Zenith, they haue no shadowe at all: or els, *Amphiscii*, hauing 2 shadowes, the one Septentrionall, when the Sunne goeth from them toward the South, the other Meridionall, whē he passeth from them toward the North.

Those whose shadowes are finite, are named *Heteroscii*, as hauing but one of those shadowes, either Septentrionall, as in the Septentrionall temperate zone, or els Meridionall, as in the Meridionall temperate zone, whereof *Lucane* maketh mention.

As concerning the inhabitants of the world, whose comparison one with another standeth vpon the Horizon, or the paralleles and Meridians, we haue 5. thinges to consider.

1. Some of them haue the same sensible Horizon, whome *Albertus* calleth *Simul habitantes*, dwelling together.

2. Some of them do dwell vnder the opposite pointes of the same parallele, and are called proprelie by the Greekes *Perioeci*, as if you would say, dwellers about, of the Latines *Transuersi*, dwellers on the other side.

3. Some of them dwell vnder the same parallele, but not in the opposite pointes, hauing a diuerse Longitude, whome *Albertus* calleth circulare dwellers.

4. Some of them dwell vnder the pointes of the same semimeridian equidistant from the Equator, hauing a contrarye Latitude, and are called *Antoeci*, or *Antomi*, also oblique inhabitantes.

5. Some of them do inhabite an equall, or also the

the same parallele, but vnder the pointes of the Meridian diametrallie opposite, and are called *Antipodes, Antichthones*, and opposite.

The distinction of the Surface of the earth, according to the length of the daies. Chap. 41.

FOr the more exact knowledge of the longest dayes in euery place of the world, sensible changing them selues, the Astronomers haue deuised, the distinction of paralleles, and of Climates.

The paralleles offer vnto our consideration, their definition, and their supputation.

They are defined to be circles distinguishing the climates, and distant one from another at the most, but quarters of howers.

Their supputation is diuerse, deliuered by 3. sortes of Geographers.

1. By the common Geographers, which do distinguishe the space of the earth from 12. gr. 45. mi. vnto 50. gr. 30. mi. into 15. paralleles, attributing vnto eache one $\frac{1}{4}$ of an hower.

2. By the Mariners, who in like manner do recon 14. paralleles, distinguished by quarters of howers, from the Equator vnto 45. gr: but then they proceede by halfe howers, vnto the 19. parallele: and then by adding on whole hower, they come vnto the 21. prrallele.

3. By the more subtile Geographers, who make 48. seuerall paralleles, from the Equator toward the pole of the world, vnto the 66. gr. 30. mi of eleuation: and from thence augmenting them

them by dayes, they adde 20. more.

The climates are to be considered in their definition, in their diuision, in their number, and in their magnitude.

The definition is thus: A climate is a space of the earth, included within 3. paralleles, conteining the difference of ½ an hower.

Their diuision, is either general, or particulare.

Their generall diuision it that, in regarde whereof some of them are called Northern climates, and some Southerne.

The Northern climates haue their propre names, deriued from the places through the which they do passe.

The Southerne climates are those that are named by the contrarie.

Their particulare diuision is that, in regarde whereof euery one of them is diuided into 3. paralleles, the first, the middlemost, and the last parallele.

Their number is knowne through the supputation of the paralleles.

Their magnitude is inequall, aswell in respect of their Longitude, as of their Latitude.

Their Longitude toward the Equator, is greater, by reason of the greater compasse of circles: and towarde the poles it is lesse, by reason of their lesse compasse.

Their Latitude is inequall, in respect of the space of degrees, that halfe an hower doth conteyne, and it is greater about the Equator, by reason of the almost equall compasse of the degrees: and lesse about the poles, by meanes of the

the narrowe inclination of the roundnesse of the earth.

Of the light, and of the shadowes, and their differences. Chap. 42.

FOrasmuch as there hath bene often mention made of the shadowes, it shall not be amisse if we set before your eyes, the methodicall description thereof: and seeing that contraries are by their contraries made more manifest, we will declare the nature of the light, and of the shadowe.

The nature of the light is shewed by the definition, the diuision, and the cause thereof.

It is defined to be the image, or the beame of the bright light.

It is diuided, either into the first and principal, or the secondarie and reflexed light.

The first and principall is that, which proceedeth directly from the light body, and is either perpendiculare, or oblique.

The perpendiculare light is that, by the fall whereof right angles are made.

The oblique light is that which falleth not at right angles.

The secondarie or reflexed light is that, which from one side spreadeth it selfe on al parts, without any falling of the beames.

The cause of the light is either the Elementall bright light, whereof here we teach nothing, or the celestiall.

The celestiall bright light, is that which either causeth the shadowe, as that of the ☉, of the ☾, of ♀: or els, which hath no power to make any shadow, as the light of the of the other Starres.

The nature of the shadowe is declared by the definition, and by the diuision thereof.

It is defined to be a light diminished: or a certen forme of a darke body, alwaies contrarie to the body casting the light.

The diuision thereof is two folde, the one drawen from the coastes of the worlde, the other from the position of the darke body.

The shadowe taking the appellation from the coastes of the worlde, is of 2. sorts.

The one is extended toward some coaste, and it is either Orientall, or Occidentall, or Meridionall, or Septentrionall.

The other is perpendiculare, or a right shadowe by a perpendiculare, which is not extended, as it is vnto those, that haue the ☉ in their Zenith.

The diuision deriued from the position of the darke bodie is that, in respect whereof one shadowe is called right or extended, another reuersed.

The right shadowe is that, which is caused by the darke bodie, perpendicularelie erected vpon the terrestriall plane.

The reuersed shadowe is that, which is caused by the darke bodie, that is parallele vnto the Horizon.

Of

Of the Eclipses in generall. Chap. 43.

OF al the apparēces of the heauen, the Eclipse is the principall: and therefore we will declare the generall reason of the same, by the definition, and by the termes thereof.

The definition is either barelye and planelye propounded, or els it is more largely expressed.

The plane definition thereof is that, whereby it is defined to be the taking away, or the hindering of the bright light, so that it cannot come vnto the eye.

The larger expression thereof is thus: vnto euery Eclipse there belong 3. thinges: a bright heauenly light, our sight, and a shadowy or dark bodie.

The bright heauenly light was formed by the Creator, for the expelling of shadowes, and it is twofolde, a greater and a lesse.

The greater, is that of the ☉, shining of it selfe.

The lesse is that of the ☾, casting about (as out of a looking glasse) her light borowed of the ☉.

Our syght is diuerse, according to the diuerse position thereof, vppon the round compasse of the earth.

The shadowy or darke bodie is also twofold, viz. the bodie of the ☾, the one moitie whereof the ☉ enlighteneth not: and the earth, whose shadow is alwaies opposite vnto the ☉.

The termes of the Eclipse, which in this kinde

of

of doctrine the Astronomers do vse, are in number three.

The first is the quantity of the bodie either of the ☉, whose visuall diameter (as a chord) doth subtende in the Auge of his eccentricke 31. mi. and in the opposite thereof 31. mi. or else of the ☾, whose apparent diameter doth in the Auge of her eccentricke and epicycle, subtende 29. mi. and in the opposite thereof 36. mi.

The secōd is the quantity of the shadow, which the motion of the ☉ through either *Absis*, doth cause to varie, aswell in regard of the longitude from the surface of the earth, which for the most part conteineth 276. semidiameters of the earth, as also in respect of the latitude, which also in the place of the ouerthwart crossing of the ☽ is diuers, both in respect of the ☉, beeing in either *Absis*, and of the ☾ which in her opposition is either in the *Auge* of her epicycle, and then it is 75. mi. or in the opposite thereof, and then it conteineth 94. mi.

The third is the quantity of the termes eclipsed, either of the ☾, which are 15. partes 12 mi. or of the ☉, by reason of the *Parallax* of the latitude of the ☾, being either about the South, and it is 11. gr, 22. mi. or about the North, being 20. gr. 40. mi.

The particular description of the Eclipses, Chap. 44.

THe beames therefore both of the ☾ and of the ☉, may be hindered from shining vpon the earth.

The beames of the ☾ being borowed, may bee hindered by the comming of the earth, and the shadow thereof, betweene the ☉ and her, and that maketh the Eclipse of the ☾, whereof wee may consider the time wherein it happeneth, & the continuance thereof.

The time of her eclipse is when shee is at the full, when the ☉, being in opposition with the ☾, driueth the shadowe either according to the longitude, as euerye moneth it commeth to passe, or els according to the latitude, whiche falleth out when the ☾ is either within or nigh vnto the *Nodi*, that is, the head and taile of the Dragon.

In the continuance it is to be considered, that the staye of the ☾ in her darkening, is either long or short.

The long stay is with her whole bodie, when her opposition falleth out precisely in the *Nodi*.

The short stay is when she is distant from the *Nodi*, and then her body is darkened either all, or halfe, or lesse then the halfe.

Shee is darkened wholly, when shee hath her latitude lesse then the semidiameter of the shadow, by the quantity of her apparent semidiameter.

Shee is darkened halfe, when she hath her latitude equall vnto the semidiameter of the shadow.

Shee is darkened lesse then the halfe, when she hath her latitude greater then the semidiameter of the shadow.

The beames of the ☉ are hindered by the interposition of ☾, and that is called the Eclipse of the ☉, wherein wee may haue consideration of the tyme wherein it happeneth, the diuersitie thereof, and the difference thereof from the Eclipse of the ☾.

The time wherein the ☉ is eclipsed, is in the new ☾, at which time she seemeth to haue a diametrall coniunction with the ☉, aswell in respect of longitude, as of latitude.

The diuersitie thereof is, in that it is eclipsed either wholly, or lesse then all:

The ☉ is wholly eclipsed, when the ☾ beeing in visible cōiunction with the ☉, is in the *Node*.

The ☉ is eclipsed lesse then wholly, when as the ☾ being in visible coniunction with the ☉, hath latitude, but yet lesse then 35. mi. or els, when the semidiameters of the ☉ and the ☾ are ioyned together.

The difference of the Eclipse of the ☉ from the eclipse of the ☾, is in regarde of the time, the continuance, and the vniuersalitie.

The difference considered in the time, is in that the ☾ is darkened in the opposition, but the ☉ in the coniunction.

The difference cōsidered in the continuance, is in that the darkening of the ☾ falleth out to be long, but the Eclipse of the ☉ but short, by reason of the small quantity of the D, and the swifte motion thereof.

The difference considered in the vniuersalitie, is in that the Eclipse of the ☾ is euery where seene, but the Eclipse of the ☉, in one onlie parte of the earth, namely in that, which is couered by the shadowe of the ☽.

FINIS.

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VIII. Of the four Compendiums for quadratique Equations.
IX. Of Recreative Problems.

By *Michael Dary.*

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TO THE READER.

Courteous Reader,

THou haſt here preſented to thy view and cenſure a few Mathematical Notes, moſt whereof have lain by me many years; and the reaſon of their ruſhing into the Publick in this homely dreſs, is, for that I and ſome others have been traduced and derided in a Book lately publiſhed, Entituled, *A Guide to the Young Gager*, put forth by ſeveral Authors: In the tail of which Book there is a whole Broad-ſide (intricate, prepoſterous, inartificial, and moſt prodigiouſly erroneous) diſgorged by the *Lieutenant* or *Bringer-up* againſt a Book, Entituled, *The Art of Practical Gauging*; which Book

I am ſorry to ſee ſo incumbred with Preſs-Faults, though they were none of mine: But moſt of theſe Guns we ſhall Charge again, and turn them upon this our *Lieutenant*, for we ſcorn to give an Anſwer to his invective Examination, till we have firſt examined him.

In his *Stereo.prop*.pag.6.pr.2.he tells how to find the Solidity of a Fruſtum Pyramide whoſe baſes are parallel but not alike: The word *Fruſtum Pyramide* I cannot underſtand, but if he had ſaid Fruſtum of a Pyramide, he would have been underſtood by every judicious man: But the Solid there ſpoken of (as I am informed) might very fitly have been called a *Priſmoid*, for it is a kind of a Priſm; for although the Sides thereof ſhould be continued, they would never be included or terminated in one point, as the Pyramide is; therefore, why Fruſtum Pyramide?

For the conſtruction of this his Fruſtum Pyramide, he bringeth (as an induction of particulars) four Pyramides, four Priſms, and a Parallelepipedon, that is in all five Priſms; (for a Parallelepipedon is a Priſm) but in reality there is but two Priſms, and the

the four Pyramides, which are indeed but one Pyramide, in the largeſt ſenſe it is capable of, both baſes being parallel but not alike, each being a rectangle.

In his Propoſition pag. 105. (which he proſecutes in pag. 106, 107, 108.) the ſtreſs of his Argument is weak and infirm (I paſs by the Preſs-Fault pag. 105. $z + 2$ for it ſhould be $z - 2$) for pag. 106. he ſaith $z = \frac{3}{5}$ that is z is 3 and A, 5; theſe numbers make good the Queſtion: But ſtay, though we ſhould grant $z = \frac{3}{5}$, it is yet to demonſtrate, that z is 3 and A, 5; which I am not bound to tell him how to do: But it is evident the man made haſt to the 108 page, to fling dirt in the face of *Van Schooten* (for he doth not care who he doth beſpatter) and it is manifeſt, if you compare this laſt page with his Title page, which ſaith, *Particularly intended for Gaging*; let any man judge, whether this Propoſition have any relation at all to Gaging. But we would have him to know, that we can perform this Propoſition and two other, of good uſe for the rational Ordinates of the Circle and Hyperbola, without caſting dirt in the face of any perſon.

 Prop.

Prop. 1. $xx+yy=zz$. quef. x? y? z?
Sol. $x=aa-bb$. $y=2ab$. $z=aa+bb$.

Prop. 2. $xy+yy=zz$. quef. x? y? z?
Sol. $x=2ab+bb$. $y=aa$. $z=aa+ab$.

Prop. 3. $xx+xy+yy=zz$. quef. x? y? z?
Sol. $x=aa-bb$. $y=2ab+bb$. $z=aa+ab+bb$.

In all thefe three Solutions you may put a and $b=$ any two numbers taken at pleafure.

Moreover in Page 107. his Note for Progreffions is invalid, and of no force: For

the Sum { of the Sum and Diffe- } greateft.
{ rence of any two num- }
{ bers is equal to the }
the Diff. { double of the ——— } leaft.

From whence the Argument is clear by affumption, for he affumes the fame greateft number twice: Therefore the Sum of the Sum and Difference muft be the fame that it was before; to wit, the double of that fame greateft number: So it is evident that

that there is no need of Unity for the firft Term of this Progreffion as he intimates: If he fhall fay his Note is true notwithftanding my declamation, then let him fhew a reafon why he doth amufe his Reader with fuch a foolery. Are thefe the Men of a found judgement which they fpeak of in their Preface? The like general Theorem may be laid down in Geometrical Progreffions, but no need of Unity for the firft Term.

The Fact { of the Fact & quote } greateft.
{ of any two numbers }
{ is equal to the }
The quote { Square of the ——— } leaft.

But this I fhall not infift upon.

In his invective Examination, Pag 1. he faith, That our Equation is five times greater than it needs: If he could have faid it had required five times more work in the operation, he had faid fomething; but to fee how full he is fraught with Envy, for this very Propofition hath been commended by divers Artifts in this City, for the contrivance of it, because it doth accommodate both the Spheroid and the Parabolical Spindle with one and the fame Di-

Divisor, and the Operation is very near as short as the old way for the Spheroid.

From the first page to the twelfth he is wholely taken up with inveighing against the Table of Segments; in which I see there are many Press Faults, but these Faults (as I said before) are none of my Faults: For the making of the Table I gave this Rule, 6,2831853):0,017453 A−S:×Q(=K, which I shall demonstrate.

It is manifest from *Archimedes*, *Snellius*, and other Authors, that if the Radius of a Circle be Unity, that then the Area of that Circle is 3,14159266 too much, or 3,14159265 too little: Also from the same Authors it holds, that the Area of a Sector is equal to the Fact or Rectangle of the Radius into $\frac{1}{2}$ the Arch or Base of the Sector: Then if you put A= to the Arch, S= to the Sine of that Arch, it must hold 1×$\frac{1}{2}$A= the Area of the Sector, and 1×$\frac{1}{2}$S= the Area of the Triangle in the Sector; but the Sector less the Triangle is equal to the Segment of the Circle in that Sector: So then $\frac{1}{2}$A — $\frac{1}{2}$S = any Segment of a Circle whose Radius is Unity.

And

And by the 11. and 12. Prop. *Partis Cyclicæ* of *Leotaud's Examen* of the Quadratures of *Greg.* of St. *Vinc.* it holds; As the Area of the Circle given 3,14159265, is to $\frac{1}{2}$ A — $\frac{1}{2}$ S; so is Q, the Quadrature or Area of any Circle proposed, to K, its like Segment. Then multiplying both the first and second terms by 2, in Jimbals (for so he calls the Symbols in derision) it will stand thus;

6,2831853) : A—S::× Q:(—K.

Now because A stands in Degrees and Centesimes, it must be reduced to the same parts that S stands in, which may be done thus; As 360 deg. the whole Peripheria in the Parts of A, is to 6,2831853 the whole Peripheria in the parts of S; so is A, to 0,017453A: therefore 0,017453A, comes in the room of A, and the Rule stands thus;

6,2831853):0,017453A—S::×Q(:=K

which was to be demonstrated.

In page 12. this insulting Scoffer saith, *This may serve for instruction to Segment-Makers for the future, to inform them whether their Work goes on regularly or not.*

Reader

Reader, I hope thou hast more under-ſtanding than to take this *Bringer-up* for one that modeſtly makes publick pure *Geometry* (as he ſpeaks in his Preface.) But if thou doſt, I ſhall undeceive thee: For it is apparently true, that if a Table of the Segments of a Circle ſhall be differenced never ſo often, they have no equal Differences; but he ſeems to intimate that they have: For if he mean otherwiſe, I would fain know of him the Habitude of that rank of Differences by which he will prove the truth of a Table of Segments. But methinks I hear the Reader object and ſay, That if a Table of Segments be differenced far enough, the Differences at laſt will be equal: Stay there, herein lies the deception; for thoſe Figures that appear to be equal, are only frontier-Figures, and if you make the Table of Segments a good company of places larger, you will then ſee the ragged Regiment that ſtands behind theſe Frontiers. This he ought to have told his Reader, otherwiſe he publiſheth (not pure, but) very impure *Geometry*.

In page 13. he proceeds to the Trial of the

the Tables of Wine, and Beer and Ale; and the Table for dividing the Gaging Rod, and he concludes them alſo badly calculated, as may be proved (ſaith he) by taking the ſecond or third Differences.

What? the ſecond, or the third? 'Tis a marvel when his hand was in, he had not put in the *fourth* Differences too: *O ye Blind Guides*! know ye not that the Tables for Wine, Ale and Beer, are capable but only of the firſt and ſecond Differences; which I prove thus for the Wine: the conſtruction of the Wine Table is the Square of any Diameter in Inches, divided by 883. Then if you put D = a Diameter propoſed (do you ſee now) in Jimbals it will ſtand thus:

	1 Diff.	2 Diff.
DD + 0D + 0		
DD + 2D + 1	2D + 1	
DD + 4D + 4	2D + 3	2
DD + 6D + 9	2D + 5	2
DD + 8D + 16	2D + 7	2

All to be divided by 883. And if you ſhould interpoſe never ſo often, you ſhall find no more but the ſecond Differences. But

But it ſeems they will have the ſecond and third Differences come in, for *'tis all one to* Anthony *who Kiſſeth* Dorothy.

But what if our *Lieutenant* ſhould ſay he did intend the third Differences for the Trial of the Gaging-Rod Table: But certainly he did not, (out of doubt he would not be ſo unkind to his *Young Gager*, to leave him thus in the dark *guideleſs*,) if he did he is caught in an evil Net, for there he might as well have ſaid the thirteenth as the third, for that Table is the ordinates of a Parabola ſtanding at equal diſtances. This may ſerve for an inſtruction to our Table-Tryers, how they burn their Fingers again: Let them learn how to be Table-Makers before they turn Table-Tryers.

Courteous Reader, I am ſorry I have held thee in this diſcourſe ſo long: Now let me addreſs my ſelf to thee, that we may underſtand one another. In Chap. 6. Prop. 1. thou wilt meet with a Solid, which I call a *Priſm*; by which word is there meant a Solid having two Baſes, equal, parallel, alike, and alike ſituate, and

and in the Peripetaſma a Right Line may be every where applied, from one Baſe to another.

A *Pyramide* of the ſame Baſe and height with the Priſm, is $\frac{1}{3}$ thereof: And in the Peripetaſma a Right Line may be every where applied, from the Baſe to the Vertex.

A *Pyramidoid* of the ſame baſe and height with the Priſm, is ſome certain portion thereof; as if it be parabolical, it is $\frac{1}{2}$; if it be ſpherical, it is $\frac{2}{3}$: And in the Peripetaſma a Right Line may be no where applied from the Baſe to the Vertex.

For my Diviſion, it is ſuch as is uſed by others: As for Example, 24) : 37 + 41 : × 4 (= 13. and may be read thus; 24 dividing 37 more 41, multiplyed by 4, equal to (or quotes) 13: Or it may be read by analogie thus: As 24, is to 37 more 41; ſo is 4, to 13. If thou meeteſt with ſome Diviſions that ſtand double lined, they were things that had lain by me a good while, and I would not ſtand to alter them.

So

So craving thy favourable construction, where any thing hath slipt amiss, for it was not the intent of him who desires, if he were able, to be

Thine to serve thee,

Michael Dary.

Errata

ERRATA.

THe *five last Pages are wrong numbered,* 42, 43, 44, 45, 46 *they should be* 44, 45, 46, 47, 48; *and then pag.* 24. *lin.* 20. *for* of its *read* of all its *pag.* 26. *lin.* 15. *for* A=S *read* A—S *p.* 29. *l.* 5. *for* L.= *read* =L. *p.* 31. *l.* 5. *for* 0,159 *read* 0,16 *p.* 33. *l.* 1. *for* If *read* 8. If *p.* 36. *l.* 12. *for* Conjugate *read* rectangular Conjugate *l.* 16. *for* rectanguled *read* rectangular *l.* 18. *for* rectanguled *read* rectangular *p.* 37. *l.* 8. *for* =D *read* =D, *p.* 43. *l.* 3. & 4. *for* sides *read* lines *p.* 45. *l.* 14. *for* let be *read* let it be, *and for* terms in *read* terms: In. *p.* 47. *l.* 16. *for* number *read* any number. *Pag.* 36. *for* Convex Begirter *or* Zone *you may read* Peripetasma.

The

The Contents.

Dary's *Miscellanies.*

CHAP. I.

Of the Inscription and Circumscription of a Circle.

1. FOrasmuch as the *Ratio* of an Arch line to a right line is yet unknown, it is absolutely necessary, that right lines be applyed to a Circle for the Calculation of Triangles wherein Arch lines come in Competition.

2. Right lines applyed to a Circle are

 Chords,

Chords, Sines, Tangents, Secants, and versed Sines.

3. The Chord of an Arch is a right line extended from one end of that Arch to the other end thereof: The Sine is a right line drawn from one end of that Arch, Perpendicularly upon the Diameter drawn from the other end of that Arch: The Tangent is a right line touching one end of that Arch extended till it Concur with the Secant: The Secant is a right line extended from the Center of the Circle till it Concur with the Tangent: The Versed Sine is a right line being a Segment of the Diameter, drawn from one end of that Arch till it be cut by a Perpendicular (i. e. the Sine) from the other end of that Arch.

4. It is to be noted by this Definition in Prop. 3. that the Chord of an Arch is common to two Arches, one of them being the Complement of the other to a whole Circle; and likewise the Versed Sine is common to two Arches, one of them being the Complement of the other to a whole Circle: But the Sine of an Arch is common to two Arches, one of them

them being the Complement of the other to a Semi-circle.

5. As the Sum of two Sines is to their difference, so is the Tangent of the $\frac{1}{2}$ Sum of those Arches, to the Tangent of their $\frac{1}{2}$ difference.

6. As the Sum of two Tangents, is to their Difference, so is the Sine of the Sum of those Arches, to the Sine of their Difference.

7. As the Sine of the Sum of two Arches, is to the Sum of their Sines, so is the difference of those Sines, to the Sine of their Difference.

8. If you put R = The Radius of a Circle, A = an Arch proposed, C = the Chord of that Arch, S = the Sine of that Arch, T = the Tangent of that Arch, and Z = the Secant of that Arch.

Then

$$\frac{S\ R}{Co,\ S} = T$$

$$\frac{R.\ R}{Co,\ T} = T$$

$$\frac{RR}{Co,S} = Z$$

$$Z + T = T \text{ of } \overset{deg.}{45} + \tfrac{1}{2} A$$

$$Z - T = T \text{ of } 45 - \tfrac{1}{2} A$$

$$2\,Z = \text{Sum} \begin{cases} T \text{ of } \overset{deg.}{45} + \tfrac{1}{2} A \\ \text{and} \\ T \text{ of } 45 - \tfrac{1}{2} A \end{cases}$$

$$2\,T = \text{Dif.} \begin{cases} T \text{ of } \overset{deg.}{45} + \tfrac{1}{2} A \\ \text{and} \\ T \text{ of } 45 - \tfrac{1}{2} A \end{cases}$$

$$\frac{RR + RT}{R - T} = T \text{ of } \overset{deg.}{45} + A$$

$$\frac{RR - RT}{R + T} = T \text{ of } \overset{deg.}{45} - A$$

$$\frac{2\,RRT}{RR - TT} = T \text{ of } 2\,A$$

$$\sqrt{:\frac{RRRR}{TT} + RR:} - \frac{RR}{T} = T \text{ of } \tfrac{1}{2} A$$

9. If

9. If twice three Arches equi-different be propoſed, Then, as the Sine of one of the means, is to the ſum of the Sines of its Extreams; ſo is the Sine of the other mean, to the ſum of the Sines of its Extreams.

10. And hence, if a rank of Arches be equi-different, As the Sine of any Arch in that rank, is to the ſum of the Sines of any two Arches equally remote from it on each ſide; So is the Sine of any other Arch in the ſaid rank, to the ſum of the Sines of two Arches next to it on each ſide, having the ſame common diſtance.

11. Three Arches equi-different being propoſed, If you put Z = the Sine of the greater extream, Y = the Sine of the leſſer extream, M = the Sine of the Mean, m = the Co-ſine thereof; D = the Sine of the common difference, d = the Co-ſine thereof, and R = the Radius.

1. Then $Z + Y = \frac{2\,Md}{R}$

2. The

2. Then $Z - Y = \frac{2\,mD}{R}$

3. Then $ZY = MM - DD$.

4. Then $\frac{Z}{Y} = \frac{Md + mD}{Md - mD}$

12. From the last before going, it is evident, that if two thirds (i.e either the former, or the latter 60 deg. or the former 30 deg. and the latter 30 deg.) of the Quadrant be compleated with Sines: the remaining third part of the Quadrant maybe compleated by Addition or Subduction onely.

13. If in a Circle, two right lines be inscribed cutting each other, The Rectangles of the Segments of each line are equal. And the Angle at the point of Interfection is measured by the Half-sum of its intercepted Arches.

14. If to a Circle two right lines be adscribed from a point without; The Rectangles of each line from the point assigned to the Convex and Concave are equall. And the Angle at the assigned point is measured by the half difference of its intercepted Arches

15. If in a Circle (or an Elipsis) three right lines shall be inscribed, one of them cutting the other two: Then the Rectangles of the Segments of each line so cut, are directed proportional to the Rectangles of the respective Segments of the Cutter.

16. If a plain Triangle be inscribed in a Circle, the Angles are one half of what their opposite sides do subtend:

17. Therefore the Angles of a plain Triangle are equal to a Semi-circle.

18. And hence, if a Rectangled Triangle be inscribed in a Circle, the Hypothenuse thereof is the Diameter of the Circle.

19. As the Diameter of a Circle is to the Chord of an Arch; so is that Chord, to the versed Sine of that Arch.

20. And hence, if from the right angle of a rectangled Triangle, a Perpendicular be let fall upon the Hypothenuse, the Hypothenuse is thereby cut according to the *Ratio* of the squares of the sides.

21. If in a Circle, any plain Triangle be inscribed, and a Perpendicular be let fall upon one of the sides, from the opposite angular point; Then as that Perpendicular is to one of the adjacent sides, so is the other

other adjacent side, to the Diameter of the Circumscribring Circle.

22. If a Circle be inscribed within a plain Triangle, Then, as the Perimeter is to the Perpendicular; so is the Base on which it falleth, to the Radius of the inscribed Circle.

23. If a Quadrilateral Figure be inscribed in a Circle, and Interfect with Diagonals, The Rectangle of the Diagonals is equal to the two Rectangles of the opposite sides.

24. If a Circle be both inscribed and circumscribed by two like ordinate Polligons; Then, as the Co-versed Sine of the side of the inscribed is to the Diameter, so is the Area of the Inscribed to the Area of the Circumscribed.

25. If an ordinate Polligon be both Inscribed and Circumscribed by two Circles; Then, as the Diameter of the Circumscribed, is to the co-versed Sine of the side of the Polligon; So is the Area of the Circumscribed, to the Area of the Inscribed.

26. In any right lined Figure, if a Circle be Inscribed; Then, as the Peripheria of the Circle, is to the Area thereof; So is the Perimeter of the right lined Figure, to the Area thereof. *Et Con.*

27. But in all Circles, as the Peripheria is to the Area, so is 2 to the Radius.

28. Therefore, In any right lined Figure, if a Circle be inscribed, as 2. is to the Radius; So is the Perimeter of the right lined Figure, to the Area thereof.

CHAP.

CHAP. II.

Of Plain Triangles.

1. A Triangle is a Figure Comprehended of three ſides; and is either Plain or Spherical.

2. A Plain Triangle is that which is deſcribed on a Plain Surface, whoſe three ſides are right lines; and it is either right Angled or oblique Angled; and the oblique, is either obtuſe or acute.

3. If a line drawn from the top or vertex of a Triangle equally Biſecting the Baſe, be equal to the Biſegment, the Vertical Angle is a right Angle; if leſſer Obtuſe, if greater Acute.

4. In a Plain Triangle, a right line equally

equally Biſecting the Vertical Angle, cuts the Baſe directly according to the *Ratio* of the adjacent legs.

5. Any one ſide of a Triangle is leſs than the ſum, and greater than the difference of the other two ſides.

6. Any one ſide being continued, the exterior Angle is equall to the two interior Angles oppoſite.

7. In any right angled Plain Triangle, the ſum of the ſquares of the ſides containing the right Angle is equal to the ſquare of the Hypothenuſe.

8. In a Plain Rectangled Triangle any one of the ſides may be put for Radius, and the other ſides ſhall be Sines, Tangents, or Secant.

9. In any plain Triangle the ſides are directly proportional to the Sines of their oppoſite Angles, *Et Con.*

10. In any Plain Triangle, as the ſum of any two ſides is to their difference, ſo is the Tangent of the Half-ſum of their oppoſite Angles, to the Tangent of their Half-difference.

11. In any Plain Triangle, as the Baſe is to the ſum of the Legs, ſo is the

the difference of the legs ; to the difference of the Segments of the Base, cut by a Perpendicular from the vertical Angle.

12. In any Plain Triangle, as the Base is to the sum of the legs ; so is the Sine of $\frac{1}{2}$ the vertical Angle, to the Sine of the sum of $\frac{1}{2}$ the vertical Angle, and either of the Angles conterminate at the Base

13. In any Plain Triangle, as the Diameter is to the versed Sine of the vertical Angle ;

So is the square of the sum of the legs } less by the square of the difference of the legs
To the square of the Base

14. In any Plain Triangle, the fact of the legs and the Sine of their Angle, is equal to the fact of the Base, Perpendicular, and Radius.

CHAP.

CHAP. III.

Of Spherical Triangles.

1. A Spherical Triangle, is that which is described on the surface of a Sphere.

2. The sides of a Spherical Triangle, are Arches of three great Circles mutually intersecting each other.

3. The measures of Spherical Angles, are arches of great Circles, described from the Angular Pointsas their Poles, and subtending their Angles.

4. Those are said to be great Circles, which bisect the Sphere.

5. Those Circles which cut each other at right Angles, the one of them passeth by the Poles of the other. *Et Con.*

6. The Distance of the Poles of two great Circles, is equal to the Angle comprehended by them.

7. The

7. The 3 Angles of any Spherical Triangle being given, there are likewise three sides of another Spherical Triangle given, whose Angles are equall to the sides of the former Triangle.

8. The sum of the Sides of a Spherical Triangle are less than two Semi-circles.

9. The sum of the 3 Angles of a Spherical Triangle, are greater than two right Angles, but less than Six.

10. Two Angles of any Spherical Triangle, are greater than the difference between the 3 Angle and a Semi-circle.

11. Any side being continued, the Exterior Angle is less than the two Interior opposite ones.

12. In any Spherical Triangle, the difference of the sum of two Angles and a whole Circle, is greater than the difference of the third Angle and a Semi-circle.

13. A Spherical Triangle, is either Rect-angular, or Oblique-angular.

14. A Rect-angular Spherical Triangle, is that which hath one right Angle at the least.

15. The Leggs of a Rect-angular Spherical Triangle, are of the same affection with their opposite Angles.

16. In a Rect-angular Spherical Triangle, if either legg be a Quadrant, the Hypothenuse is also a Quadrant; but if both be of the same affection, the Hypothenuse shall be less than a Quadrant; if of different affections, then greater: *Et Con.*

17. In a Rect-angular Spherical Triangle, if either of the Angles at the Hypothenuse be a right Angle, the Hypothenuse shall be a Quadrant; but if both of the same affection, it shall be less, if different then greater: *Et Con.*

18. In a Rect-angular Spherical Triangle, either of the Oblique-angles is greater than the Complement of the other, but less then the difference of the same Complement to a Semi-circle.

19. An Oblique angular Spherical Triangle, is either Acute-angular, or Obtuse-angular.

20. An Acute angular Spherical Triangle, hath all its Angles Acute.

21. An Obtuse angular Spherical Triangle, hath all its Angles Obtuse or mixt,

mixt, *viz.* Acute and Obtuse.

22. In an Acute angular Spherical Triangle, each side is less than a Quadrant.

23. In an Oblique angular Spherical Triangle, if two Acute Angles be equal, the sides opposite to them shall be less than Quadrants; if Obtuse, greater.

24. In an Oblique angular Spherical Triangle, if two Acute Angles be unequal, the side opposite to the lesser of them shall be less than a Quadrant; but if Obtuse, the side opposite to the greater, shall be greater.

25. In every Oblique angular Spherical Triangle, if the Angles at the Base be of the same affection, the Perpendicular drawn from the top of the Vertical Angle shall fall within the Triangle; if different, without.

In Oblique angular Spherical Triangles, if a Perpendicular be drawn from the Vertical Angle, to the opposite side, (continued if need be.)

CONsectary 1. The Co-sines of the Segments of the Base are directly proportional to the Co-sines of the sides of the Vertical Angle: *Et Con.*

Con. 2. The Co-sines of the Angles at the Base are directly proportional to the Sines of the Vertical Angles: *Et Con.*

Con. 3. The Sines of the Segments of the Base are reciprocally proportional to the Tangents of the Angles Conterminate at the Base: *Et Con.*

Con. 4. The Co-sines of the Vertical Angles are reciprocally proportional to the Tangents of their sides: *Et Con.*

Axioms for the Solution of Spherical Triangles.

Axiom 1.

IN Rect-angular Spherical Triangles having the ſame Acute angle at the Baſe: The Sines of the Hypothenuſes are proportional to the Sines of their Perpendiculars.

Axiom 2.

In Rect-angular Spherical Triangles having the ſame Acute angle at the Baſe: The Sines of the Baſes and the Tangents of the Perpendiculars are proportional.

Axiom 3.

In all Spherical Triangles, the Sines of the Angles are directly proportional to the Sines of their oppoſite ſides: *Et Con.*

Axiom 4.

In all Spherical Triangles, as the fact of the ſides containing the Vertical angle, is to the ſquare of the Radius; So is the fact of the Sines of the ½ ſum, and the ½ difference of the Baſe and difference of the Legs, to the ſquare of the Sine of ½ the Vertical Angle.

Or thus,

In all Spherical Triangles, having added all the 3 ſides together, find the difference betwixt each ſide and their half ſum: And then,

As the fact of the Sines, of the ½ ſum of all the ſides, and the difference of the ſide oppoſite to the Vertical Angle, is to the fact of the Sines of the differences of the containing ſides from the ſaid ½ ſum, ſo is the ſquare of the Radius to the ſquare of the Tangent of ½ the Vertical Angle.

CHAP. IV.

Of the Projection of the Sphere in Plano.

1. *Orthographically.*

IF a Sphere be by a Plain cut into two Hemiſpheres, and the Eye be placed at an infinite diſtance, Vertically to one of the Hemiſpheres; then a right line infinitly extended from the Eye, to any aſſigned point in the Spherical ſurface of that Hemiſphere, ſhall porject the aſſigned point upon the Plain: And the diſtance upon the Plain, from the apex of the Hemiſphere to the projected point, is equal to the ſine of the Arch from the Vertex of the Hemiſphere to the aſſigned point; the Radius of the Sphere being put for Radius.

2. *Steriographically.*

If a ſphere be by a plain touch't, and the eye be placed in the Spherical-ſurface Diametrically oppoſite to the touch-point; Then a right line infinitly extended from the eye to any aſſigned point in the Spherical ſurface ſhall project the aſſigned point upon the Plain: And the diſtance upon the Plain from the touch-point to the projected point is equal to the Tangent of ½ the Arch from the touch-point to the aſſigned point: The Diameter of the Sphere being put for Radius.

3. *Gnomonically.*

If a Sphere be by a Plain touch'd, and the Eye be placed at the Center of the Sphere; Then a right line infinitly extended from the Eye to any aſſigned point, in the Spherical ſurface (whoſe diſtance from the touch-point is leſs than a Quadrant) ſhall project the aſſigned point upon the Plain: And the diſtance upon the plain from the touch-point to the projected point is equal to the Tangent of the Arch from the touch-point to the assigned point: The Radius of the Sphere being put for Radius.

To project a Meridian line upon any Horizontal plain.

1. Having prepared a piece of Metal or Wood, and made it a true Plain, and in some convenient point thereof (taken as a Centre) erected a Gnomon of sufficient length, at right angles to the Plain: This done, fix the Plain truly Horizontal

2. If you take the Suns Co-altitude (i. e. his distance from the Zenith) three times in one day, and (according to the Steriographical projection (having a line of Tangents by you) set off from the Centre of your plain or foot of the Gnomon, the Tangent of ½ each Arch upon his respective Azimuth, or shadow (continued if need be) made by the Gnomon at that instant when the Co-altitude is taken, you will insert three points upon the Plain.

3. If

3. If you find out the Centre to those three inserted points, then a rigth line infinitely extended by this Centre found, and the Centre of the Plain, or foot of the Gnomon, is the true Meridian line: Which was to be projected.

CHAP. V.

Of Planometry, and the Center of Gravity.

1. IN any plain Triangle, the fact of the Base, and the Perpendicular, is equal to the double of the Area of that Triangle.

2. In any Plain Triangle, as the Diameter is to the Sine of any one of the angles, so is the fact of the adjacent Legs to the Area.

3. And hence, in how many soever Plain Triangles, having one angle equal in

in Common, the facts of their ſides including the common Angle, are directly proportional to their Areas. *Et Con.*

4. In any plain Triangle, as the fact of the Diameter and the Sine of any one Angle, is to the ſquare of the oppoſite ſide; ſo is the fact of the Sines of the other two Angles to the Area.

5. If you put P = the Semi-perimeter of any plain Triangle, and D, $\mathcal{D}$, d, = the reſpective differences accruing by Subduction of each perticular ſide from the Semi-perimeter, and the Area = A, Then :

1) √ D $\mathcal{D}$ d P (= A.

6. A Triangulate (i. e. any right lined figure) is Compoſed of Triangles, and the Triangles are leſs by two than the number ſides, and the Diagonals are leſs by three : And the Area thereof is equal to the Area of its Triangles.

7. If you put 1 = the Radius of a Circle, then the Area (and alſo the Semi-peripheria) ſhall be = 3,1416 *fere* : according to *Van Culen*, *Snellius*, and *Hugenius*.

8. If you put D = the Diameter of a Circle, P = the Peripheria, and A = the Area, then :

1,27324) D D (= A

12,5637) P P (= A

1) 3,1416 D (= P

1) √ : 12,5637 A : (= P

3,1416) P (= D

1) √ : 1,27324 A : (= D

9. If you put K = the Area of the Segment, or Kant of a Circle; V = the Verſed Sine in the Segment, D = the Diameter of the Circle, or Q equal to its Area; alſo if you put D) 2 V (= u, a Verſed Sine (to be found in a Table of Verſed Sines, the Diameter being 2,000 *&c.*) whoſe reſpective Arch in Degrees and Decimals being doubled, you may call A, and the correſpondent Sine of A you may call S, Then :

8) : 0,017453 A — S : × D D (= K

Or

6,2831853) : 0,017453 A — S : × Q (= K

10. Again, if you put *K* or k = the greater or lesser Segment of a Circle, cut by a Chord-line, D = the Diameter of the Circle y = the difference of the Segments of the Diameter cut at right Angles by the foresaid Chord-line, also if you put D) y (= S, a Sine (to be found in the Common Table of Sines, the Radius being 1,0000 *&c.* :) whose respective Arch in Degrees and Decimals being doubled you may call A, and the correspondent Sine of A you may call S, Then :

8) : 3, 1416 + 0,017453 A + S: × DD (= *K*.

8) : 3, 1416 — 0,017453 A = S: × DD (= k.

Another way :

1, 27324) : 0, 5 + 0, 00277 A + 0, 16 S : × DD (= *K*.

1, 27324) : 0, 5 — 0, 00277 A — 0, 16 S : × DD (= k.

11. If you put Z = the Area of a Zone of a Circle intercepted between the Diameter and Chord-line parallel to it; D = the Diameter of the Circle, B = the breadth of the Zone, Then :

10D — 15 B) : 10D — 16B : × BD (> Z.

10D — 16 B) : 10D — 17B : × BD (< Z.

12. If you put k = the Area of the Segment of a Circle (not greater then a Semi-circle,) C = the distance of the Centre of Gravity from the apex of the Segment, r = the Radius, c = the Chord of the Segment, and u = the Versed Sine in the Segment, Then :

9 r — 6 u) : 6 r — 4 u : × c u (< k.

9 r — 6 u) : 6 r — 3 u : × c u (> k.

25 r — 15 u) : 10 r — 6 u : × u (< G.

25 r — 15 u) : 10 r — 5 u : × u (> G.

13. If you put H = the Area of an Hyperbola, G = the distance of the Centre of Gravity from the apex of the Hyperbola, a = the Axis, B = the Base, r = the

the Semi-tranſverſe Diameter (between the Vertex of the Hyperbola, and the Center of the aſſymtoptes) Then :

$$9r + 6a) : 6r + 4a : \times Ba (> H.$$

$$9r + 6a) : 6r + 3a : \times Ba (< H.$$

$$25r + 15a) : 10r + 6a : \times a (> G.$$

$$25r + 15a) : 10r + 5a : \times a (< G.$$

14. If you put D = the diſtance of the Center of Gravity of a Plain right lined Triangle from one of the Angular points, and l = the right line from that Angular point Biſecting the oppoſite ſide, Then: $3) 2 l (= D$.

15. If you put D = the diſtance of the Center of Gravity of the Sector from the Center of the Sector, c = the Chord, r = the Radius Biſecting A = the Arch, Then: $3A) 2rc (= D$.

16. If you put L = the diſtance of the Center of Gravity of a, (= the leſſer of two Superficial figures propoſed) and l = the diſtance of the Center of Gravity of A, (= the greater of two Superficial Figures

Figures propoſed) from the common Center of Gravity of both the foreſaid Figures, and D = the whole diſtance of their reſpective Centers of Gravity, Then :
$A + a) DA (L.$ = and $A + a) Da (= l.$

17. As an unite is to the Radius, ſo is the exceſs of the 3 angles above a Semicircle (in a Spherical Triangle) to the boſſed ſurface of that Triangle. The exceſs is to be taken in the ſame parts, as is the Radius.

18. If a Sphere be encloſed in a Cylinder, and that Cylinder be cut with plains parallel to its baſe; then the Intercepted rings of the Cylinder are equal to the Intercepted ſurfaces of the reſpective Segments of the Sphere.

CHAP. VI.

Of Solid Geometry.

1. If you put Pr = A Priſm, B = one of its Baſes, and P = the Perpendicular

pendicular height of the Prisme, Then :
1) B P (= Pr.

2. If you put Fr = the Frustum of a Piramide or a Cone intercepted between two Plains Paralel cutting the axis, B = the greater Base, b = the lesser Base, and P = the Perpendicular height of the Frustum Then,
3) : B -|- √ B b -|- b : × P (= Fr.

3. If you put Fr = the Frustum of a Spherical Piramidoid, Sphere, or Spheroid, Intercepted between two Plains parallels, one of them passing by the Centre, B = the greater Base, b = the lesser Base and P = the Perpendicular height of the Frustum, Then :
3) : 2 B -|- b : × P (= Fr.

4. If you put Fr = the Frustum of a Porabolical Piramidoid or Conoid Intercepted between two Plains parallel, cuting the axis, B = the greater Base, b = the lesser Base, and P = the Perpendicular height of the Frustum : Then
2) : B -|- b : × P (= Fr.

5. If you put S = a Sollid made by Rotation, R = the Radius or nearest distance, between the Centre of Gravity of

A

A = the begetting Figure, and a right line in the same Plain assigned (without) for an axis, and p = the Peripheria of a Circle whose Radius is unity, Then :
1) A R p, (= S. or Thus 0,159) AR) = S.

6. If you put Fr = the Frustum of a Parabolical Spindle, intercepted between two plains Parallel, one of them passing by the Centre, B = the greater Base, b = the lesser Base, P = the Perpendicular height of the Frustum, Then :
15) : 8 B -|- 7 b : × P (= Fr.

7. In the 2, 3, and 4 Propositions : (which Propositions are general for Piramids or Cones, Piramidoids or Conoids, of what Base soever) If it will serve your turn to find onely the Cone, Conoids, and Parabolical spindle, when their Bases are Circles, it may be delivered thus,

1. If you put Fr = the Frustum of a Cone incepted between to Plains parallel, cutting the Axis at right Angles, D = the Diameter of the greater Base, d = the Diameter of the lesser Base, P = the Perpendicular hight of the Frustum, and make S = D -|- d, Then :
3,82) : S S — D d : × P (= Fr.

2. If

2. If you put Fr = the Fruſtum of a Sphere or Spheroid, intercepted between two Plains paralel, one of them paſſing by the Center Cutting the Axis at right Angles, D = the Diameter of the greater Baſe, d = the Diameter of the leſſer Baſe, and P = the Perpendicular height of the Fruſtum, Then :

$3,82) : 2\,DD + dd : \times P (= Fr.$

3. If you put Fr = the Fruſtum of a Parabolical Conoid, intercepted between two Plains paralel cutting the Axis at right Angles, D = the Diameter of the greater Baſe, d = the Diameter of the leſſer Baſe, and P = the Perpendicular height of the Fruſtum, Then :

$2,54) : DD + dd : \times P (= Fr.$

4. If you put Fr = the Fruſtum of a Parabolical Spindle, intercepted between two Plains Paralel, one of them paſſing by the Center cutting the Axis at right Angles, D = the Diameter of the greater Baſe, d = the Diameter of the leſſer Baſe, and P = the Perpendicular height of the Fruſtum, Then :

$19,1) : 8\,DD + 7\,dd : \times P (= Fr.$

If you put Fr = the Fruſtum of a Sphere intercepted between two Plains parallel, one touching and the other cutting the Sphere, d = the Diameter of the Baſe; and P = the perpendicular height of the Fruſtum, Then :

$2,54648) : dd + 1\tfrac{1}{3}\,PP : \times P (= Fr.$

This Rule will alſo hold if it were the fruſtum of a Spheroid, putting dd = the fact of the right angled Conjugates in the baſe.

9. If you put Fr = the fruſtum of a Cone intercepted between two Plains parallel, one of them being fixed the other moveable, D = the Diameter of the fixed baſe, p = the perpendicular height of the fruſtum, and d = the increment or decrement of any two Diameters at one Inch diſtance in the perpendicular, Then :

$3,82) : 3\,DD \pm 3\,Ddp + ddpp : \times p (= Fr.$

10. A Cooper's common Cask, that is ſuch as are round at their heads (and not eliptical as ſome Oyl Cask are) being

Propoſed, if you put D = the Diameter at the bouldge, d = the Diameter at the heads, G = the diagonal from the middle of the bongue-hole to the bottom of either of the heads, L = the length of the Veſſel, and make S = D + d, Then :

$L = \sqrt{: 4GG - SS :}$

$G = \frac{1}{2} \sqrt{: LL + SS :}$

$S = \sqrt{: 4GG - LL :}$

$D = S - d.$

$d = S - D.$

11. According to the Equated Circle, now in uſe, if you put Q = the Quantity of Liquor in a Coopers common Cask being filled totally or partly, the axis being poſited parallel to the horizon, the Veſſel being taken as the middle Fruſtum of a parabolical Spindle intercepted between two Plains parallel, equidiſtant from the Centre cutting the axis at right angles, D = the Diameter of the bouldge, V =

V = that proportion of this Diameter which is wet, d = the Diameter at the heads, and P = the perpendicular height or length of the Veſſel, alſo you ſhall put D) V (= N, which N abuts you to K = a Segment to be found in the Table of the Segments of a Circle whoſe Area is unity: Or if you have not by you a Table of Segments you may find K by *Ch. p.* 5. *Prop.* 9. and then if you divide by 19,1 you ſhall have cubical Inches, if by 4412, Wine Gallons; if by 5386, Ale or Beer Gallons; and the Rule will ſtand thus :

$19{,}1) : 8DD + 7dd : \times P \times K (= Q.$

But if the ſaid Veſſel be taken as the fruſtum of a Spheroid intercepted, &c. Then inſtead of $: 8DD + 7dd :$ you ſhall put $: 10DD + 5dd :$ And yet the foreſaid Diviſors hold true to all intents and purpoſes.

12. Concerning the *Cylindroid*, &c. in its ſeveral kinds and ſeveral fruſtums.

By the word Cylindroid (in this place) is meant a Sollid contained under three

Surfaces *(i. e.)* two Plains parallel and a Convex begirter, whereof the two Plains parallel are called the Bases, and are both Circles or both Ellipsis, or else one a Circle and the other an Ellipsis, and the Convex Surface is called the Zone; in which Zone there may be every where a right line applyed from any point in one base to some point in the other: and if such a Cylindroid be cut with two Plains meeting in the Centers of both Bases, cutting (or rather inserting) conjugate Diameters in both Bases, Then:

If you put C = the sollid Content of a Cylindroid.

A & B aloft = the two rectangled conjugate Diameters.

G & H below = the two rectangled conjugate Diameters.

A & G opposite = the two correspondent Diameters.

B & H opposite = the two correspondent Diameters.

Also P = the perpendicular height of the Cylindroid, Then:

$3,82)\overline{:A+\frac{1}{2}G:\times B+:G+\frac{1}{2}A:\times H:\times P}(=C.$

13. To cut the Cylindroid with divers Plains parallel, the Plains being of one common distance, and that distance being taken for Unity in the lineal Mensuration of the Cylindroid: To do this from that Base wherein G and H are Conjugates, you shall make P) A — G (= D and P) B — H (= d, and Then:

$:3G+1\frac{1}{2}D:\times H+:1\frac{1}{2}G+D:\times d =$ first Fr

$:3G+4\frac{1}{2}D:\times H+:4\frac{1}{2}G+7D:\times d =$ first diff.

$3DH+:3G+12D:\times d =$ second diff.

$6Dd =$ third diff.

All to be divided by 3,82.

If it shall so happen that A = B and G = H, it is the frustum of a Cone, Then:

$3GG+:3G+D:\times D =$ first Fr.

$3GG+:9G+7D:\times D =$ first diff.

: 6 G + 12 D : × D = second diff.
6 D D = third diff.
All to be divided by 3,82.

If it shall so happen that B = H, Then:
: 3 G + 1 ½ D : × H = first Fr.
: 3 G + 4 ½ D : × H = first diff.
3 D H = second diff.
All to be divided by 3,82.

If it shall so happen that B = H = G, Then:
: 3 G + 1 ½ D : × G = first Fr.
: 3 G + 4 ½ D : × G = first diff.
3 D G = second diff.
All to be divided by 3,82.

If it shall so happen that A = B = G = H, Then it is a Cylinder: to be divided by 3,82.

14. Now instead of P = the perpendicular of the whole height of the Solid, if you shall put p = the perpendicular height of any part thereof, from that Base wherein G and H are Conjugates, and C = the solid content of the Cylindroid at that height,

height, then it holds as in the margin:

$C = (p \times : Dd\,pp + p \times : 1\frac{1}{2}Gd + 1\frac{1}{2}HD + 3GH :: 3,82)$

But if such a solid have not its Zone made by Circles or Ellipsies but by four flat sides at right angles to the foresaid Conjugates, then it is a *Prismoid*: Neverthelesſ the Rules before prescribed hold to all intents and purposes, if you take away the Divisor 3,82 and in the room thereof place the Divisor 3.

15. This last Proposition to find the Content of the Cylindroid or Prismoid at any height or depth may be also performed by the Table of figurate Numbers following, thus:

1. Having got the first Frustum, first, second, and third differences (if there be so many) of your Solid, multiply them by the respective numbers in the Table at that height or depth.
2. Add all these Products together into one Sum, having respect to the signs + and — (if they shall be so signed) and this Total is the Content of your Solid at the height or depth proposed.

A

A Table of Figurate Numbers for the ſpeedy Collection and exact Correction of the firſt, ſecond and third Differences, in finding the Content of the Cylindroid or Priſmoid.

Depth.	1 Fruſt.	1 Diff.	2 Diff.	3 Diff.
01	1	00	000	0000
02	1	01	000	0000
03	1	02	001	0000
04	1	03	003	0001
05	1	04	006	0004
06	1	05	010	0010
07	1	06	015	0020
08	1	07	021	0035
09	1	08	028	0056
10	1	09	036	0084
11	1	10	045	0120
12	1	11	055	0165
13	1	12	066	0220
14	1	13	078	0286
15	1	14	091	0364
16	1	15	105	0455
17	1	16	120	0560
18	1	17	136	0680
19	1	18	153	0816
20	1	19	171	0969
21	1	20	190	1140
22	1	21	210	1330
23	1	22	231	1540
24	1	23	253	1771
25	1	24	276	2024
26	1	25	300	2[illegible]00
27	1	26	325	2600
28	1	27	351	2925
29	1	28	378	3276
30	1	29	406	3654
31	1	30	435	4060
32	1	31	465	4495
33	1	32	496	4960
34	1	33	528	5456
35	1	34	561	5984
36	1	35	595	6545
37	1	36	630	7140
38	1	37	666	7770
39	1	38	703	8436
40	1	39	741	9139
41	1	40	780	9880
42	1	41	820	10660
43	1	42	861	11480
44	1	43	903	12341
45	1	44	946	13[illegible]44
46	1	45	990	[illegible]4[illegible]90
47	1	46	1035	[illegible]5[illegible]0
48	1	47	10[illegible]	16[illegible]
49	1	48	11[illegible]	[illegible]
50	1	49	1[illegible]	[illegible]

3. The construction of this Table is very easie, for you may see it is nothing but a Collection of Unites: But to make the respective numbers in this Table for the first, second, and third Differences to any Depth proposed, without any gradual Collection, this is the Rule.

If you put D = the Depth

Then 1) $D - 1$ (= 1 Diff.

2) $DD - 3D + 2$ (= 2 Diff.

6) $DDD - 6DD + 11D - 6$ (= 3 diff.

Or if you put D = the first Diff.

Then 1) D (= 1 Diff.

2) $DD - D$ (= 2 Diff.

6) $DDD - 3DD + 2D$ (= 3 Diff.

16. The symetry of like Superficies and like Solids. Superficies are like, and Solids are like, if the Angles be equal in number and quantity,

17. Like

17. Like Superficies are one to another as the Squares of their Correspondent Sides: Like Solids are one to another as the Cubes of their Correspondent Sides.

CHAP. VII.

Of the Construction and Use of the Scale of Ponderosity (commonly called) the Still-yard.

1. A Right Line resting on a fulciment equiponderate being proposed: Then if any ponderosity shall be applyed to a point of Pendency in that Line; it ought to be understood, that that Ponderosity is transplanted from the Fulciment to that point of Pendency: But if any Ponderosity shall be withdrawn or taken away from a point of Pendency in that Line, it ought

ought to be underſtood that that Ponderoſity is tranſplanted from the point of Pendency to the Fulciment.

2. A Right Line reſting on a Fulciment equiponderate being propoſed : If divers Ponderoſities ſhall be pendantly applyed on ſundry points of that Line , ſo that the ſaid line be equiponderate again : Then the facts (of each Ponderoſity by its tranſplantation from the Fulciment) on this ſide the Fulciment , are equal to the facts on that ſide the Fulciment.

3. A Right Line, reſting on a Fulciment, equiponderate by divers Ponderoſities, pendant in ſundry points of that line, being propoſed: If two of the Ponderoſities pendant ſhall be tranſplanted, ſo that the ſaid line be equiponderate again : Then the facts of each Ponderoſity by his diſtance run in tranſplantation are equal.

4. If a Stillyard or Scale of Ponderoſity, or (as the *Dutch* call it) the *Roman* Beam, be true , (*i. e.* doth give the truth) in two points (the farther diſtant the better) it is true in all the points of pendancy throughout , . the Diviſions being equal.

CHAP.

CHAP. VIII.

Of the four Compendiums for quadratique Equations.

IN theſe four Compendiums you have both the affirmative and negative Roots ſymbolically expreſt , z being the unknown Symbol in each Equation, (but made known by each Solution) a being the known factor in the firſt term ; b being the known factor in the ſecond term ; r being the reſolvend.

But if at any time an Equation ſhall be propoſed, incumbred with vulgar fractions, let be reduced to its leaſt terms in whole numbers, if poſſible ; if not , let it be reduced to its leaſt terms in Decimals ; and then it will fall under one of theſe four Equations following.

In which Equations you muſt note, that if there be no known factor expreſt in the firſt term , then a is underſtood to be unity : Furthermore, it is evident, that if any quantity

quantity ſhall be ſigned — that then t[he] the Square Root, or the Root of any ev[en] power of ſuch quantity ſo ſigned, is ine[x]plicable: Therefore when this ſhall hap[pen] in the Solution of any Equation wha[t]ſoever, that Equation may be ſaid to [be] impoſſible.

1. Equa. $+azz+bz=+r$

Sol. $2a)\ \pm\sqrt{:+bb+4ar:}\ -b\ (= {+z \atop -z}$

2. Equa. $+azz-bz=+r$

Sol. $2a)\ \pm\sqrt{:+bl+4ar:}\ +b\ (= {+z \atop -z}$

3. Equa. $+azz-bz=-r$

Sol. $2a)\ \pm\sqrt{:+bb-4ar:}\ +b\ (= {+z \atop +z}$

4. Equa. $+azz+bz=-r$

Sol. $2a)\ \pm\sqrt{:+bb-4ar:}\ -b\ (= {-z \atop -z}$

CHAP.

CHAP. IX. *Of Recreative Problems.*

Problem 1. *To find* a, b, c, *three numbers under this qualification.*

$aa=bb+cc$. queſ. a? b? c?

Sol. $A=nn+1$. $B=nn-1$. $C=2n$.

You may put $n=$ any number.

Probl. 2. *To find* a, b, c, *three numbers under this qualification.*

$ab+bb=cc$. queſ. a? b? c?

Sol. $A=2n+1$. $B=nn$. $C=nn+n$.

You may put $n=$ any number.

Probl. 3. *To find* a, b, c, *three numbers under this qualification.*

$a+ab+bb=cc$. queſ. a? b? c?

Sol. $A=2n+1$. $B=nn-1$. $C=nn+n+1$.

You may put $n=$ number.

Probl. 4. *To find* a, b, c, d, e, *five numbers under this qualification.*

$a+b=c$. $cc-a=dd$. $cc-b=ee$.

Queſ. a? b? c? d? e?

Solution $A=\dfrac{nnnn+4nnn+5nn+2n}{:nn+4n+1:(2)}$

$B=\dfrac{2nnn+5nn+4n+1}{:nn+4n+1:(2)}$

C=

$$C = \frac{nn + 2n + 1}{nn + 4n + 1}$$

$$D = \frac{nn + n}{nn + 4n + 1}$$

$$E = \frac{n + 1}{nn + 4n + 1}$$

You may put $n =$ any whole number. But note, that every Equation is to be reduced to its least terms, if need require.

Probl. 5. *To find the Content of any Sollid made by Rotation, if you can get the* ratio *of the Squares of its Ordinates, to any rank of Rectangles: This is the* Theorem.

If two ranks of Quantities or Numbers be proposed (in any qualification or order whatsoever) having one common *ratio* between each pair of Correspondents: Then in both ranks their Correspondent Sums or Differences have the same common *ratio*. This Theorem holds as well in Superficies if you can get the *ratio* of the Ordinates to any other rank of Quantities or Numbers.

FINIS.

GAUGING EPITOMIZED:

OR,

An Abbreviation of Solid Geometry, *concerning the Business of* Cask-Gauging, *taking a* Cask *in any of the Four Notions following.* *By* Michael Dary.

PREMONITION.

If you put D = the Diameter of the Bouldge, d = the Diameter of either of the Heads, and y = D − d; L = the length of the Axis of the Vessel, and C = the Content thereof, the dimension being taken in Inches.

Prop. 1. If a Cask be taken as the middle Frustum of a *Spheroid*, intercepted between two Planes parallel cutting the Axis at right angles: Then, $3,82) : 2DD + dd : \times L \;(= C$.

Prop. 2. If a Cask be taken as the middle Frustum of a parabolick Spindle, intercepted between two Planes parallel, cutting the Axis at right angles: Then, $3,82) : 2DD + dd - 0,4yy : \times L \;(= C$.

Prop. 3. If a Cask be taken as the middle Frustum of two parabolick Conoids abutting upon one Common Base, intercepted, *&c.* Then, $3,82) : DD + dd : \times 1\frac{1}{2}L \;(= C$.

Prop. 4. If a Cask be taken as the middle Frustum of two Cones abutting upon one Common Base, intercepted, *&c.* Then, $3,82) : DD + dd - \frac{1}{3}yy : \times 1\frac{1}{2}L \;(= C$.

Now if you would have the Content in Beer Gallons, you must multiply 3,82 by 282, makes your divisor = 1077 *fere*: If you would have Wine Gallons, $3,82 \times 231$ makes your divisor = 883 *fere*. And from these two divisors, you may Calculate Tables for Wine or Beer Measure: For the square of any diameter in Inches divided by 883, is the Construction of the Wine Table; or by 1077, is the Construction of the Beer Table: And either of these Tables have their second differences equal, therefore they will be made by an easie Collection.

¶ *If a* Cask *be not full (the* Axis *being posited parallel to the* Horizon*) to find the quantity of Liquor contained in it.*

To do this, you ought to have a Table of the *Segments* of a Circle, whose *Area* is unity, the *Diameter* being divided into 10000 equal parts, and then this Approximation is the readiest hitherto used, which requireth this *Data*; the whole Content of the Cask, the *Diameter* at the Bouldge, and the wet Portion thereof; and the Proportion runs thus: As the whole *Diameter*, is to its wet Portion; so is the *Diameter* in the Table, (*i.e.* 10000,) to its like Portion: Which being sought in the Table of *Segments*, abutts you to a *Segment*, by which if you multiply the whole Content of the Cask, the Product is the Content of the remaining Liquor in the Cask.

Here followeth an Account of some Addenda *to, and Faults escaped in,* Dary's *Miscellanies.*

Page 4. *might have been added,* or T)RZ—RR(=T of ½A. *p.* 11. *l.* 10. *for* right anguled *r.* rectangular. *l.* 14. *for* rectanguled *r.* rectangular. *p.* 14. *pr.* 7. *should run thus,* The 3 Angles of any Spherical Triangle being given, there are likewise 3 sides of another Spherical Triangle given, whose Angles are equal to the sides of the first Triangle, if you take the Complement to 180 *deg.* of one of them, but most conveniently of the greatest. *p.* 19. *l.* 1. *for* fact of the sides *r.* [illegible] By this means all Circles of are likewiſe 3 ſides of another Spherical Triangle given, whoſe Angles are equal to the ſides of the firſt Triangle, if you take the Complement to 180 *deg.* of one of them, but moſt conveniently of the greateſt. *p.19.l.1.for* ſact of the ſides *r.* fact of the ſines of the ſides. *p.20.after the orthographical Projection ſhould have followed,* By this means all Circles of the Sphere perpendicular to the Plane, are projected into Right Lines, thoſe parallel to it into Circles of the ſame bigneſs, and all other into Ellipſes. *So after the Steriographical Projection,* By this means, all great Circles of the Sphere, paſſing thorough the Eye and Touch-point, are Projected into Right Lines, and all other into Circles, and the Angles in this Projection are equal to their Correſpondent Angles on the Sphere. *So after the Gnomonical Projection,* By this means, all great Circles on the Sphere perpendicular to the Plane, are Projected into Right Lines, all thoſe parallel to it into Circles, and all others into *Hyperbola's*, *Parabola's* or *Ellipſes*, accordingly as they cut or touch that Great Circle of the Sphere which is parallel to the Plane, or do neither. *P.23.l.15.for* Legs *r.* Sides. *P.25.l.2.& 4.for 12,5637 r. 12,5664. P.27.l.1. for* and Chord *r.* and a Chord; *l.4.for* —15B *r.* —16B. *P.28.after Prop.15 ſhould have followed,* The *Area* of a *Parabola,* is $\frac{2}{3}$ of the circumſcribed *Parallelogram* of the ſame Baſe and Height; and the diſtance of its Centre of Gravity from its Vertex towards its Baſe, is $\frac{3}{5}$ of its Altitude: Moreover in a *Semiparabola* the Centre of Gravity, is in a Right Line parallel to the Baſe, at $\frac{3}{5}$ of the Height; and in a Right Line parallel to the Axis at $\frac{3}{8}$ of the Baſe. *P.19. Prop.18. ſhould run thus,* If a Sphere be inſcribed in a Cylinder, and that Cylinder be Cut with Planes at Right Angles to its Axis, then the intercepted Surfaces of the Cylinder are equal to the intercepted Surfaces of the reſpective Segments of the Sphere. *P.31. l.1. for* the begetting *r.* the Area of the begetting; *l.13. for* 8B+7b *r.* 8B+4√Bb+3b. *P.32.l.* ult. *and P.35.l.14.& 17. for* 8DD+7dd *r.* 8DD+4Dd+3dd. *P.35. Prop.12. ſhould run thus,* By the word *Cylindroid* is meant a Solid contained under three Surfaces *(i.e.)* two Planes parallel and a Peripetaſma, whereof the two Planes parallel are the Baſes, and are both Circles or both Ellipſes, or elſe one a Circle and the other an Ellipſis, and a Right Line extended from the Centre of one Baſe to the Centre of the other, may be called the Axis of the Solid; and in the Peripetaſma a Right Line may be any where applied from Baſe to Baſe, being in the ſame Plane with the Axis of the Solid.

But if ſuch a Cylindroid be Cut with two Planes meeting in the Centers of both Baſes, Cutting (or rather Inſerting) rectangular Conjugate Diameters (or Axes) in both Baſes, then *&c. P.37.l.2. for* divers *r.* many. *P.39.l.3.for* Zone *r.* Peripetaſma; *l.6. for* to the foreſaid Conjugates *r.* between themſelves, then it is a Priſm or Priſmoid. *P.42.l.* ult. *for* quantity *r.* quantity, and their ſides proportional.

This Table of Wine Meaſure ſhould have come in Pag. 35. *of the* Miſcellanies.

The Names of Wine Veſſels.	Cubical Inches.	Pints.	Quarts.	Gallons.	Runlets.	Hogsheads.	Tercions.	Pipes or Butts.	Tuns.
1 Tun	58212	2016	1008	252	14	4	3	2	1
1 Pipe or Butt	29106	1008	0504	126	07	2	$1\frac{1}{2}$	1	
1 Tercion	19404	0672	0336	084	$04\frac{2}{3}$	$1\frac{1}{3}$	1		
1 Hogſhead	14553	0504	0252	063	$03\frac{1}{2}$	1			
1 Runlet	04158	0144	0072	018	01				
1 Gallon	00231	0008	0004	001					
1 Quart	$00057\frac{3}{4}$	0002	0001						
1 Pint	$00028\frac{7}{8}$	0001							
1 Cubical Inch	00001								