COMPUTING MACHINE
FOR THE
SOLUTION OF LARGE SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS

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It is the main purpose of this paper to present a description and exposition of a computing machine which has been designed principally for the solution of large systems of linear algebraic equations. In this description an effort will be made to strike an advantageous mean between vague generalities and the confusing detail of actual working plans. In order to accomplish this end the language will necessarily be somewhat schematic and functional, but it is hoped that this will aid rather than hinder an evaluation of the practicability of the present design. The introductory part of this paper will also contain an outline of the utility which a successful machine of this kind will have and a short resume of the development of this machine. A financial statement is appended which shows the source of funds and other less tangible aids which the project has received, the way that these considerations have been expended and a proposed budget for the future.

Utility. In the treatment of many mathematical problems one requires the solution of systems of linear simultaneous algebraic equations. The occurrence of such systems is especially frequent in the applied fields of statistics, physics and technology. The following list indicates the range of problems in which the solution of systems of linear algebraic equations constitutes an essential part of the mathematical difficulty:

1. Multiple correlation.
2. Curve fitting.
3. Method of least squares.

4. Vibration problems including the vibrational Raman effect.
5. Electrical circuit analysis.
6. Analysis of elastic structures.
7. Approximate solution of many problems of elasticity.
8. Approximate solution of problems of quantum mechanics.
9. Perturbation theories of mechanics, astronomy and the quantum theory.

This list could be expanded very considerably, for linear algebraic systems are found in all applications of mathematics which possess a linear aspect. It is interesting to notice that the applications of linear algebraic systems to practical problems are of two kinds. If a finite set of basic elements can be found so that the mathematical equations isomorphic with physical objectivity constitute a finite algebra, the application is direct and exact and is generally characterized as algebraic. If, on the other hand, no finite set of basic elements can be found but an infinite set must be used, the mathematical equations form an infinite algebra and the problem is likely to be referred to as one in analysis. In such cases there is often no way to arrive at an exact solution and the finite systems of linear algebraic equations form part of some approximate method of solving the problem. Items 1 to 6 in the above list are usually of the former type; items 7, 8, 9 are of the latter type. The writer is of the opinion that such approximate methods using large systems of linear algebraic equations constitute the only practical method of solving many problems involving linear operational equations. (This general type of equation includes differential and integral equations as special cases.) This point of view is well substantiated by examining the literature.

In this way the solution of large systems of linear algebraic equations constitutes an important part of mathematical applications. The theory and method of solution of such systems is well known and, if the number of unknowns is small (the number of equations is generally but not always equal to the number of unknowns), this solution presents no difficulties. However the satisfactory treatment of many problems, certain cases of each item of the above list for instance, requires the use of ten or more unknowns. In the approximate methods it is the general rule that to secure a closer approximation more unknowns must be used. Now it is easy to see that the principal term in the amount of labor needed to solve a system of equations is kN^3 in which N is the number of unknowns and k is a constant. Since an expert computer requires about eight hours to solve a full set of eight equations in eight unknowns, k is about 1/64. To solve twenty equations in twenty unknowns should thus require 125 hours. But this calculation does not take into effect the increased labor due to the greater chances of error in the larger systems and hence the situation is much worse than this. The solution of general systems of linear equations with a number of unknowns greater than ten is not often attempted. But this is precisely what is needed to make approximate methods more effective in the solution of practical problems.

The machine described in this paper has been designed to fill this need. It is the hope of the writer that eventually some sort of computational service can be provided to solve systems of equations accurately and at low cost for technical and research purposes.

Development of the idea. About seven years ago the writer began to investigate the feasibility of mechanizing this solution. He was aware of the possibility of using a mechanical or electrical analogue but discarded this method, as being too inaccurate and cumbersome when many equations are to be solved, in favor of a method employing direct calculation of the results desired. An examination of the theory of such systems of equations and of the isomorphic theory of matrices and determinants shows there is only one practical method of procedure, the well known process of successively eliminating one variable between pairs of equations until an answer is obtained. Cramer's solution in terms of determinants does not furnish a better computational approach to the problem because one is then faced with the evaluation of the determinants, which is as difficult a problem as the solution of the original system of equations. Computers rarely employ determinants in the solution of linear systems of equations; in fact, it is easy to show that the work is best arranged in the old form.

After the writer had formulated the general outline of a plan for mechanizing this elimination process, an attempt was made to realize this process by using the computational capacity of the commercial punched-card tabulating equipment. Rather complete details were worked out for this application but it was finally abandoned, principally because the computation capacity of even these machines is not large enough. The construction of a machine, from the beginning, of sufficient capacity seemed too involved to be attempted with the available resources.

However the solution of this problem seemed so important for practical applications that work was not abandoned. Considerable thought was given to the design of a computing mechanism that would simultaneously be simple, fast, and accurate. After many attempts to devise a conventional computing mechanism with these properties attention was turned to the possibility of changing the base of the numbers in which the computation is carried out. For a short time the base one-hundred was thought to have some promise but a calculation of the speed of computations carried out in terms of this base showed it to be so low as to make its use out of the question. However this same calculation showed that the base that theoretically gives the highest speed of calculation is e, the natural base. But the base of a number system must be an integer, and a further calculation indicated that the bases two and three yield number systems with the same and consequently the highest applicable speed of calculation. The choice of the base for a system of numbers to be used for mechanical calculation is a rather different question than if the numbers are to be used in mental calculation. In the latter case one would choose the largest base for which the computer could easily remember the multiplication tables. The base ten is well established and numbers to this base are satisfactory for mental computation although numbers to the base twelve would probably be better.

However, for mechanized computation the base two shows a great superiority. We have seen that in speed of operation base-two machines possess a theoretical advantage over machines using any other base except three and in this case the theoretical speeds are equal. Even a cursory examination indicates that computing mechanisms using base two are much simpler than those using any other base. This advantage may seem at least partly balanced by the fact that the base two required $\log_2 10 = 3.34$ times as many places to work to the same accuracy as the conventional machines using the base ten; however we shall see, in the final design, how it is possible to build machines to the base two (and perhaps to other bases) with only an insignificant increase in cost and complexity for additional places. The use of the base two in the contruction of computing machines shows a further advantage if it is desired to incorporate in the machine a device for making a record of the results of calculation in such a form that it may be used to reinsert the numbers in the machine. This result is usually accomplished by having the machine control the punching of a card. When it is desired to reinsert the number in the machine the punchings on the card are used to mechanically or electrically control the machine and so reinsert the number. Now at each spot on the card there are two possibilities; either there is a hole or there is no hole. This corresponds exactly to the use of base-two numbers and greatly simplifies the mechanism for punching the card and reading it. Furthermore if there are b possibilities at each spot on the card it is easy to demonstrate that the use of numbers to the base b permits the card to carry the maximum amount of information. In the present case this means that a card of a certain size used with the base-two recording system will carry more than three times as much data as if used with the conventional base-ten system.

It is, of course, not the purpose of the writer to promote the general use of the base-two system of numbers but this would perhaps be feasible in a highly mechanized civilization. In this case all tables could advantageously be mechanized for this would be much easier than with the present number system. Even with actual present conditions, under which it will probably be necessary to make parts of every set of calculations in terms of numbers to the base ten, it seems highly desirable to change to numbers in the base two in making extended mechanized calculations such as in the solution of large systems of linear algebraic equations. This is particularly the case since it will be shown to be a very simple matter to incorporate in the design of the machine a mechanized transformation table which will automatically transform the numbers between the two bases and the operator will not be burdened by the process.

After obtaining these results an effort was made to devise a computing mechanism operating with the base two which would combine, to the fullest possible extent, simplicity, speed and accuracy. Most base-ten computers have wheels which, in the operation of the machine, assume one of ten positions thus giving the value of the digit in that place. For brevity we shall call these wheels abacus elements. The analagous elements in base-two computers will have to be able to assume two positions; to put the matter more generally it will have to be an element capable of assuming two states corresponding to the two values 0 and 1 which the digit in that position can assume. Several forms were considered for this abacus element:

1. Purely mechanical one-dimensional systems with two positions of equilibrium, moved from one position to another with a mechanical force.

2. A moving reed like the armature of a polarized relay, moved by electromagnetic action.

3. A small piece of retentive ferromagnetic material, the two states being the directions of magnetisation of the material, this condition is changed by a strong magnetic field.

4. Certain circuits involving vacuum tubes in which there are two conditions of stable equilibrium. (The scale of two counter used in counting cosmic ray pulses has a circuit of this type.)

5. A small electrical condenser, the two states being directions of charge.

It is obvious that this element will need to be a simple inexpensive affair since it will have to be repeated thousands of times in a machine of any size. At first the fifth type of element was omitted from consideration, because the state of charge was considered too transitory a phenomenon to be used for holding numbers in the machine. However, after many forms of each of the others listed had been considered and in many cases tried without yielding an element that was satisfactory as regards cost, simplicity or readiness with which it could be interrelated with the rest of the mechanism, attention was turned to the possibility of using a small condenser. Then the idea occurred to the writer that it might be possible to so arrange the mechanism that the machine would jog its memory at short intervals, and this idea has been incorporated along with the use of condensers as abacus elements in the present design. The condensers will retain their memory (i.e. their charge) to a sufficient degree for five minutes but they have it jogged (i.e. they are recharged in their original direction) at intervals of perhaps a second.

The next problem was the design of the computing mechanism itself. The magnitude of the charge on small condensers fits in naturally with the use of thermionic tubes since this charge is large enough to control the grids of these tubes but small enough to be taken from the plate circuit of the tube. The first plans made were to use the circuit of the scale-of-two counters but after months of experimental work this idea was abandoned because of the inherent instability of the circuits. At times these circuits could be made to work but obscure factors strongly influenced their operation. At last the writer hit upon another type of circuit that in the end proved very stable and entirely satisfactory in other ways. This circuit operates upon new principles in the computing art, principles that are rather analogous to the function of the human brain in mental calculation. The circuit takes cognizance of what is in a given abacus element, what is to be added into or subtracted from the element, and from a memory device it receives a signal indicating carry over from the previous place. Having been taught by a man with a soldering iron it selects the right answer and replaces what is in the counter by this result. The computing mechanism, being a vacuum-tube circuit, operates at such speed that it can be used over and over again to add the various digits, and additional places only require the use of additional abacus elements, that is,additional condensers. At the same time the over-all complexity and cost of the computing machine is greatly reduced.

Many other details of the developmental work that has led to the final design could be given but it is felt that the past pages give some idea of the scope and duration of these efforts.

DESCRIPTION AND EXPOSITION OF MACHINE

The actual description of the machine and the explanation of its operation will be preceded by two sections of an introductory character. The first will introduce a terminology for describing the functional parts of any ordinary calculating machine (for instance the Monroe) in such a general way that it will later be useful in describing the new machine. The second will give the method used to mechanize the solution of systems of linear algebraic equations. Since the solution of such systems is the principal objective of the present design, the details of construction will be clearer with an explanation of the method of accomplishing this end in mind.

1. General Principles of Computing Machines.

A principal component in the construction of any computing machine is a set of n elements, each capable of assuming b positions or states in which b is the integral base of the number system that the computing machine is designed to employ. The number n is determined by the accuracy which is desired of the computing machine. We shall call such a set of elements an abacus. It is clear that within the limits set by n we can adjust the abacus to represent any number of base b. In an Arabic number system of base b a number P is represented by the form $\sum p_i b^i$, $0 \leq p_i < b$. The t-th element of the abacus a_t is set in the state P_t.

The usual computing machine contains two or more abaci and in almost all cases these differ in function and structure. This differentiation comes about in the following way. It is of course essential that computing machines combine numbers as well as represent them. The typical operation of combination is to add or subtract the number in one abacus to or from that contained in the other. The numbers in the two abaci thus play different roles, one is left unchanged, the other is enhanced or diminished by the first in the course of the operation. It is not essential that the abaci be of different structure, it would in fact be an advantage in a versatile computing machine if their roles could be reversed at will. However, economy generally requires the greater specialization of structure. The one that is unchanged will be called the keyboard abacus (ka) and the one that is changed will be called the counter abacus (ca). The ka often carries the additional feature that it can be set with the fingers, the ca often carries visible figures so that the results that it contains can be read with the eye; but these are special features of calculating machines since these ends are sometimes accomplished in other ways.

The coaction of the two abaci is normally not direct but through the agency of a third structure which will be called an add-subtract mechanism (asm). This asm must bring the abacus elements into coaction with a specific correspondence, add or subtract the one from the other and take care of carry-over or borrowing as the case may be.

Multiplication (division) is carried out by successive additions (subtractions). To avoid having to repeat the operation of the asm more than b-1 times, arrangements are made to shift the correspondence between the elements of the ka and the ca. Thus if the normal correspondence is $ka_i \leftrightarrow ca_i$ the correspondence $ka_i \leftrightarrow ca_{i+m}$ (i, m integers) multiplies by b^m the number added in by a single action of the asm. The exact way that the shift is used in multiplication and division is rather obvious and will not be described.

One point of interest is the following. The average number of actions of the asm needed to multiply a number P by a number X of s places is $s(b-1)/2$. Since the number of places in a number X is $s = \log X/\log b$ the number of operations required is $(b-1)\log X/2\log b$ in terms of X and b. By the use of this formula it is easy to show that the number of separate operations is 2.79 times as great for base-ten numbers as for base-two numbers. This indicates one fundamental advantage of the base-two number system in machine calculation.

2. Method of Mechanizing Solution of Linear Algebraic Systems.

We can now understand how systems of equations can be solved by the use of computing machines of large capacity. Let the system consist of N equations in N unknowns and suppose the machine contains N+1 ca's and N+1 ka's and that the individual elements of either are indicated by such notation as a_{ij} ($i=1$---$N+1$; $j=1$---n), the first index thus indicating which abacus, the second which element. The coefficients and constant term of one equation are placed on the ca's, the corresponding coefficients and constant term of another equation on the ka's. Suppose the correspondence is drawn so that

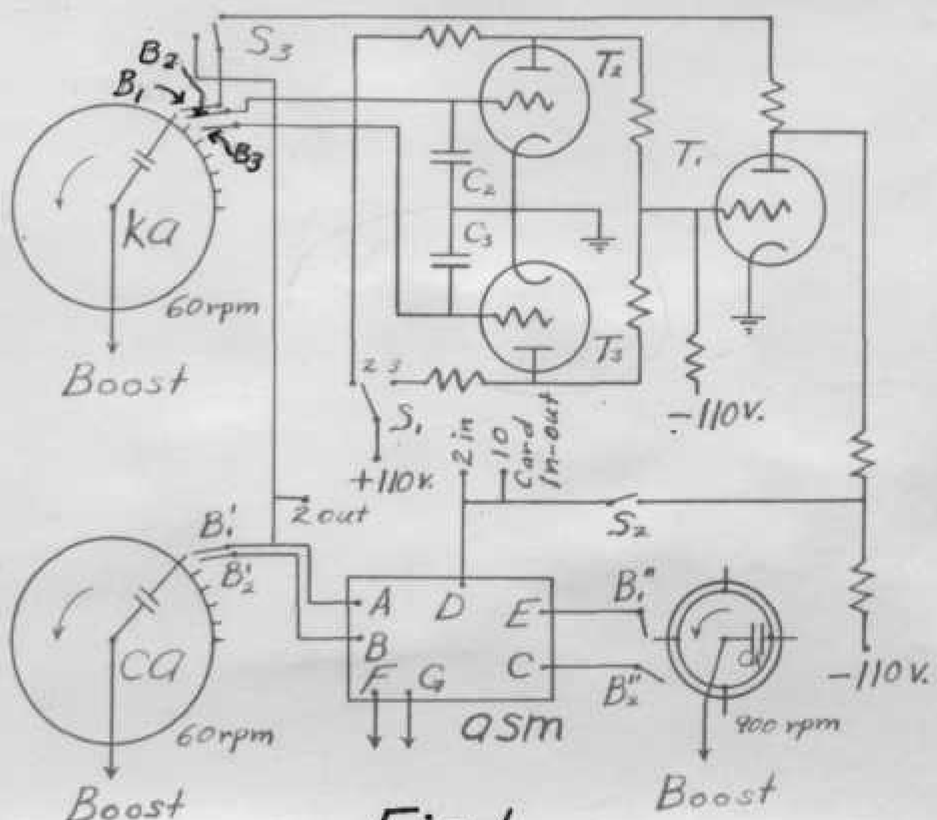
$$ka_{ij} \leftrightarrow ca_{ij+m} (i=1,\text{---}N+1,\ j=1\text{---}n)$$

and an action of the asm takes place. It is easy to see that the resulting numbers on the ca's are the coefficients and constant term of a new equation linearly dependent on the original two. However, one may select a process of repeated actions for various values of m that reduces one of the coefficients (i.e., the number on one of the ca's) to zero. (In this process, like division, the remainder on the machine is reduced to zero). The resulting numbers are the coefficients and constant term of the equation obtained by eliminating one unknown between the original two. By obvious repetitions of this process the number of unknowns is reduced to one, giving the value of this unknown. By other eliminations between this final result and previous results all unknowns can be determined.

3. Essential Elements of Computer.

The computing machine under construction is arranged to employ numbers with base of two (b=2). The abacus elements 50 in number (n=50) are small tubular paper condensers of .0015 mfds. capacity. These condensers are arranged to occupy radial positions within a hollow cylinder of bakelite. Their inner terminals are connected to a common lead, the outer terminals are connected to contacts which pass through the cylinder wall. (Fig. 8). The condensers which form the elements of one abacus and their contacts are arranged in a plane perpendicular to the cylinder axis, and are spaced at 6° intervals so that the 50 contacts occupy about 300° of the complete circle. The arrangement is so compact that 32 abaci of this size (480 base-ten place capacity) are placed in each bakelite cylinder, (8" diameter, 11" long). According to present design only 30 abaci are used at one time, the others are reserved as spares to be used if any condensers fail. Two such sets of abaci are provided, one for the ca's and one for the ka's. A positive charge on the outer end of a condenser corresponds to zero, a negative charge to 1. The magnitude of the charge does not matter except that the resulting negative potentials must not be less than the five volts necessary to block the vacuum tubes employed. The condensers are charged to about -50 volts, lose one-half their charge in perhaps 5 minutes, and thus the abaci can hold their numbers for a considerable length of time without attention. However, we shall see that their charges are renewed at intervals of one second in the present design.

The cylinders containing the ka's and the ca's rotate upon a common


S3
B2
B1
B3
T2
T1
C2
C3
T3
ka
60rpm
Boost
S1
+110V.
-110V.
2 in
10
Card
In-out
2 out
S2
B'1
B'2
A
D
E
B
C
F
G
B''1
B''2
-110V.
ca
60rpm
asm
900 rpm
Boost
Boost
Fig. 1

axis at a speed of 1 r.p.s. (Fig. 10). Brushes bear upon their contacts to read the charges on the condensers and recharge them. This is shown schematically in Fig. 1 which shows one ka and one ca along with the circuits necessary to keep the condensers charged and to permit the ka to compute into the ca. Three brushes bear on the ka and two on the ca. To see how the condensers of ka are kept charged we observe that a condenser contact touches the brushes in order B_3 B_2 B_1. Condensers C_3 and C_2 are small compared with the abacus condensers and become charged practically to the same potential; however, the triode T_3 is without plate potential and the charge on its grid is inconsequential. If the given condenser in ka is negative T_2 is blocked, thus raising the potential of its plate. This high plate potential is applied to the grid of T_1 through the network, making this tube pass. The plate potential of T_1 drops from about 120 to 50 volts as a consequence. The condenser C_2 causes this state of affairs to remain in effect at least until the next condenser terminal strikes B_2 (6° later in the rotation of ka). Meanwhile, (3° later) the condenser contact touches B_1. The lead connected to the common condenser terminals marked Boost is alternately at 0 potential when a condenser contact is touching B_2 or B_3 or at + 90 volts when a similar contact is touching B_1 ($\angle B_1, B_2 = 3°, \angle B_2, B_3 = 6°$). So in this case the condenser receives a charge of −40 v. (negative terminal outward). Likewise, if a condenser on passing B_2 is positive it will receive a charge of + 50 v on passing B_1.

The shift is accomplished by the switch S_1. When it is moved to position 3 it is easy to see that the numbers on the condensers are moved forward (in the direction of motion) one place. This electrical method of shifting is a great advantage over changing the angular relation between the cylinder and the brushes. The switch would normally be moved from posi-

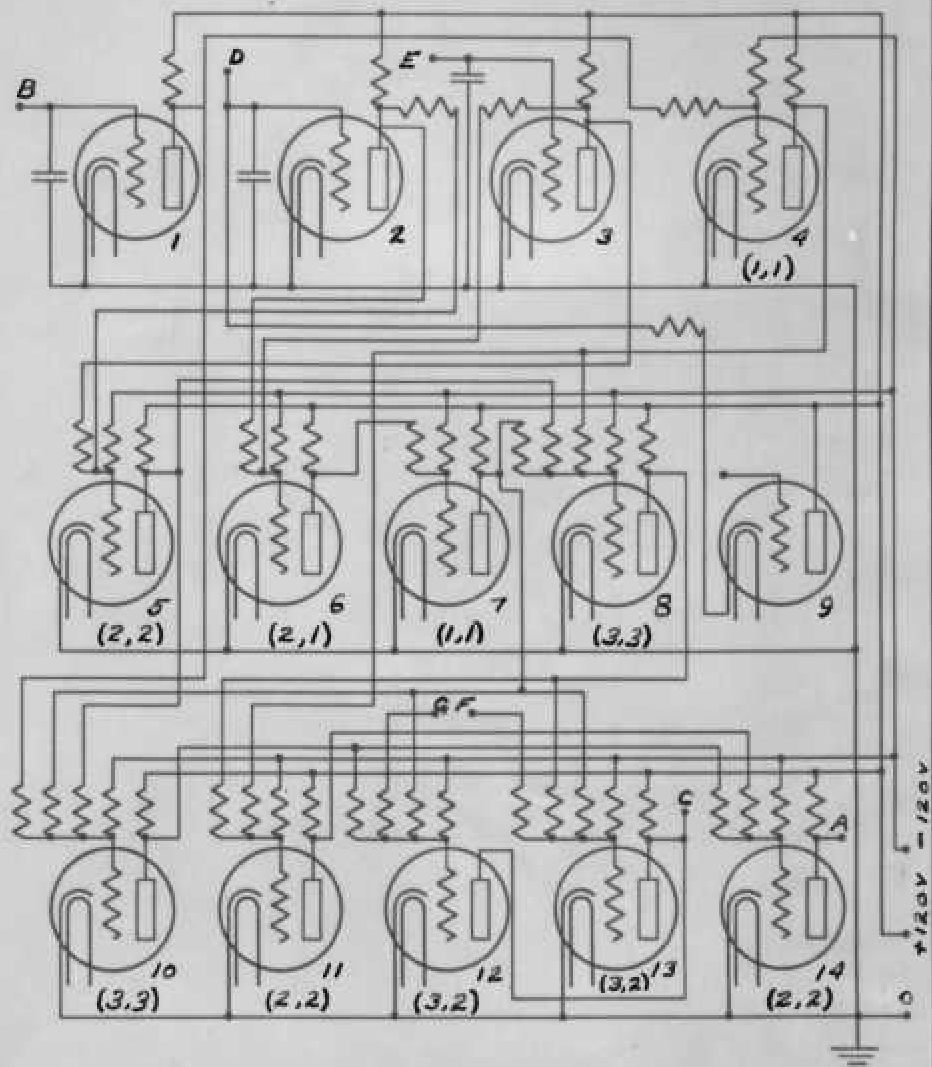

B
D
E
1
2
3
4
(1,1)
5
(2,2)
6
(2,1)
7
(1,1)
8
(3,3)
9
G F
C
A
10
(3,3)
11
(2,2)
12
(3,2)
13
(3,2)
14
(2,2)
+120V
−120V
0

Fig. 2

tion 2 to 3 during the vacant part of the cycle (i.e., when the part of the cylinder without condensers or contacts is passing the brushes) and hold there for one cycle (one revolution of the abaci cylinder) only. The output T_1 immediately shows the shift and retains it because of the shifted position of the numbers on ka.

If S_2 is closed there is a connection from T_1 through a resistance network to a point D on the asm. As the plate of T_1 swings from 50 to 100 volts the point D takes on a smaller swing, say from a value of less than -5 volts to a value greater than zero. The points B and C take on similar values, B being controlled by the condenser touchihg B_2 and C by the carry-over condenser C^4. The asm unites these pulses, charging the condensers of ca at B_1 with a number equal to the sum of the numbers on ka and ca if F is high and G low, and the difference in the opposite case. At the same time the carry-over condenser C^4 is charged according to the carry-over required to make the results correct.

Exactly how this is accomplished can be seen by reference to Fig. 2 which gives the details of the connections of the asm. The letters given correspond to those of Fig. 1. No attempt will be made to describe in detail the way the circuit reacts to each of the sixteen possible conditions with which it must contend but the following considerations will enable the reader to follow through any particular case.

The vacuum tubes used in the circuit are twin triodes, (type 6C8G). In the drawing the triodes are indicated separately but only seven envelopes are needed for a single asm. Each triode shifts between two extreme states in the operation of the circuit: the grid in the range from less than -5 volts to a value greater than zero, the plate in the range from 120 volts to a value less than 50 volts when a plate resistor of 50,000 ohms is used. (It is found to be advantageous to slightly alter the plate resistor at certain points of the circuit but this is a refinement not needed in an explanation of the circuit).

There are two principal types of circuits in which the tubes are used. The triodes 1, 2, and 3 have their grids connected to small condensers and their potentials do not change until a new charge is placed on these condensers (the triode 2 can also be discharged through the agency of the triode 9). On the other hand, the tubes 4, 5, 6, 7, 8, 10, 11, 12, 13, 14 have resistance networks across their grids consisting of p equal resistors connected to various plates and one resistor connected to the -120 v bias supply. A symbol of the type (p,q) will be noticed beside each of the triodes in Fig. 2. This indicates that there are p plate connections to the grids of this triode and that the bias supply resistor is adjusted so that at least q plates will have to be low, i.e., less than 50 volts, to block this triode. Experience as well as theory shows that when p=3, the largest number used, that the grid of the given tube changes about 12 volts when one of the plates connected to it changes from low to high. This has proved adequate to make the action of the device sharp and clean since a grid swing of only 5 volts is needed. Within a considerable range (which can be easily increased if necessary) the tube characteristics and the values of resistors and other circuit elements make no difference in the results obtained. Comparable remarks are applicable to any computing machine.

By following the diagram and making use of the (p,q) symbols attached to the various triodes, it is easy to verify the following table. In this table H stands for the higher voltage in the range and L for the lower.

Numbers Presented	Corresponding Voltage	Addition F-high G-low	Subtraction F-low G-high
B D E	B D E	A C	A C
0 0 0	H H H	H H	H H
0 0 1	H H L	L H	L L
0 1 0	H L H		
0 1 1	H L L	H L	H L
1 0 0	L H H	L H	L H
1 0 1	L H L	H L	H H
1 1 0	L L H		
2 1 1	L L L	L L	L L

The symbols A and C represent the two output terminals. If A is high the corresponding abaci element carries a zero, if A is low a one. Likewise if C is low there is a carry-over into or a borrowing from the next higher place.

In this way the machine rapidly performs the elementary operations of addition and subtraction. Other operations are carried on much as in other calculating machines, the chief difference being that the electrical principle allows great versatility of operation and control. It will be noted that there are no mechanical oscillations at the computing frequency and so full advantage may be taken of the high speed of the electrical vacuum-tube circuits.

4. Base-Two Card Mechanisms

Purpose of base-two cards. In using the machine it is highly desirable to have available means for recording the results of intermediate steps of a computation and for reinserting these results as required. Strictly speaking, such a mechanism is only an auxiliary part of the machine, and not a part of the actual computing devices; however, it is an essential auxiliary if an over-all speed of operation consistent with the speed of computing is to be obtained.

Since computations are carried on with base-two numbers it is logical to record intermediate results as base-two numbers. Also, for reasons pointed out in the introduction, a punched-card system of recording, such as here contemplated, permits more than three times the information to be carried on a given card if base two rather than base ten is used. Only one row of hole-possibilities is required for any number, making it possible to express thirty 50-place base-two numbers, the maximum carried by the computing abaci, on a single card of reasonable size.

New system of recording data. At the outset it was decided that conventional methods of mechanical punching were not suitable for this machine, as a tentative punching rate of 60 holes per second was deemed necessary. (This rate corresponds to the rate of revolution of the abaci, 50 places in 5/6 seconds, and also enables convenient use to be made of the 60-cycle power-line frequency, as will be seen later). Accordingly, attention was paid to the possibility of punching holes by electrical discharges, making use of the fact that certain dielectrics, once punctured in any spot, pass current through such a spot with only 1/3 or 1/4 of the original break-down voltage applied, therefore permitting easy read-back of information thus recorded.

A great amount of experimentation was carried on to perfect the electrical punching and reading systems, using various modifications of vacuum-tube-controlled spark coils, radio-frequency-initiated arcs, and thyratron-controlled discharges. All of these arrangements were found to function. However, from an engineering standpoint the last appears to be the most suitable, and this will be described in detail.

One other point which must be considered in addition to the high punching rate is that the mechanism must be capable of producing accurately located holes. This is required because the numbers on a card must be kept in perfect synchronism with the numbers on the abaci; that is, the first places in the numbers on a card must appear at the punching (or reading) point at the same time that the first places on the abaci appear at the first brush set, and likewise for all subsequent places. This necessitates gearing the card-handling mechanisms to the abaci shaft in order that a fixed phasal relationship and proper speed be maintained. The design calls for sets of rolls to drive the cards and a cam-driven feed-in device, which mechanisms are well known in the computing machine art and appear to be fairly straightforward.

The circuit shown in Figure 3 is used to obtain and control a punching voltage.

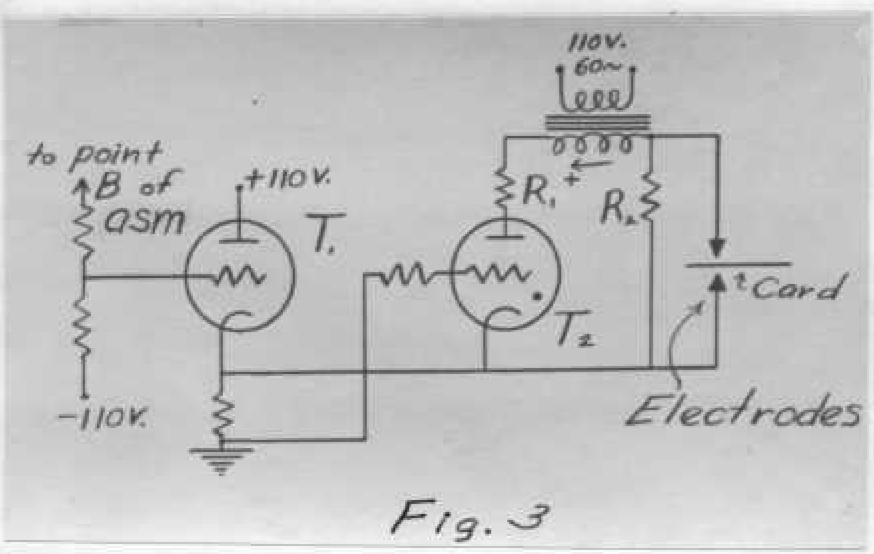


Fig. 3

The transformer supplies a peak voltage of 5,000 volts. The phase of this voltage is so adjusted that it is near its peak in the indicated direction at the instant when the first brush set is touched by a row of contacts on the abaci. The thyratron T_2 is normally held blocked, and is controlled by the corresponding computer element through an intermediate tube T_1. If the reading brush touches a negatively charged contact the point x changes in the negative direction, blocking T_1 and allowing the thyratron to ionize. Almost the entire peak voltage of the transformer immediately appears across the card, which breaks down at once and allows an arc to be formed. The purpose of R_1 is to limit the resulting current. The purpose of R_2 is to stabilize the circuit just before the tube ionizes; otherwise there would be two open circuits in series, with a possible indefinite voltage distribution. The arc automatically extinguishes itself approximately one quarter of a cycle after beginning because the voltage wave passes through zero at this time. These points are illustrated in Figure 4.

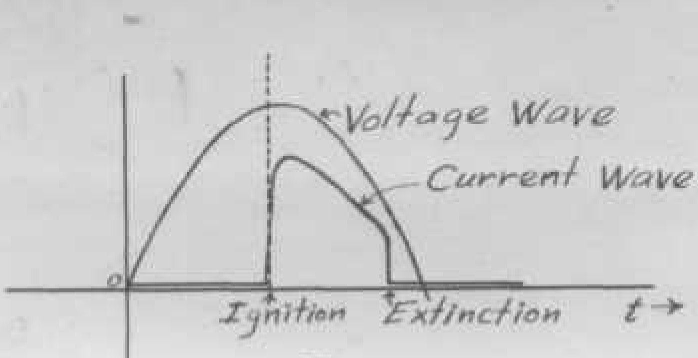


Fig. 4

It might be thought that with the card moving at the rate of 8 or 10 inches per second the holes would be lengthened in the direction of motion; however, this is not the case, since the arc is pulled along by the motion of the card and continues to pass through at the point where the break-down first occurred.

Thirty such circuits are required, one for each abacus. However, a common transformer may be used. It has also been found possible to use high-vacuum tubes in similar circuits, though thyratrons seem to be more suitable.

Card reading. The process of removing information from a card is called reading. This is a comparatively simple process when the card has been punched electrically, since it merely requires the testing in proper sequence of each possible hole-position with a voltage which is low enough not to puncture the card normally, but high enough to force an appreciable current through any point which has been broken down previously. The ratio of the punching voltage to the reading voltage may be made as high as three or four with the proper card material, thus precluding the possiblity of an accidental puncture by the reading voltage.

In reading, the card is moved along between electrodes by exactly the same sort of mechanisms as used for punching. Each set of electrodes is connected as shown in Figure 5 to its corresponding asm.

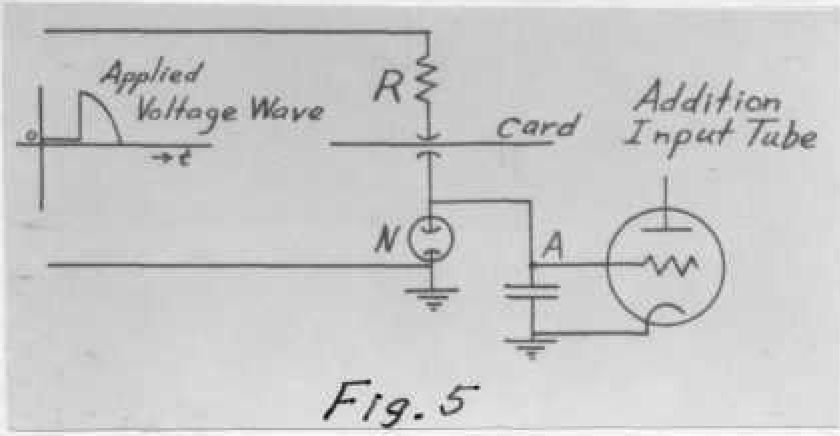


Fig. 5

The voltage is thrown on at the proper time by means of a thyratron and has the wave shape shown in the diagram in order to make more precise the timing of the reading impulses. The resistance R serves to limit the current, and the neon bulb N prevents the voltage across condenser c from becoming too high. It is apparent that should a hole exist between the electrodes, a negative impulse, corresponding to a one, enters the addition input section of the asm. As with the punching device, the same voltage source suffices for all thirty sets of electrodes.

5. Base ten Conversion

In order to facilitate entry of original information and removal of final results by the operator it is desirable to have automatic conversion within the machine between the base-two and the base-ten number systems. Three distinct devices are required: (1) a base-ten card-reading device, (2) a base-ten keyboard, and (3) a set of base-ten dials for recording final results. In conjunction with these a mechanical conversion table connecting the two number systems is required; this will be described first, then the other devices.

Conversion table. Any number is represented by a set of electrical charges on the abacus condensers. Suppose the number ten is to be placed on an abacus; this means that all condensers except the last four must be positive (zero) and that the last four be charged as follows: - + - +, that is, 1010. This can be accomplished by rotating with the abacus a device carrying two contact studs which are so positioned that the first touches a brush when the second place on the abacus is being recharged, and the second touches the brush when the fourth place on the abacus is being recharged. Figure 6 illustrates this schematically.

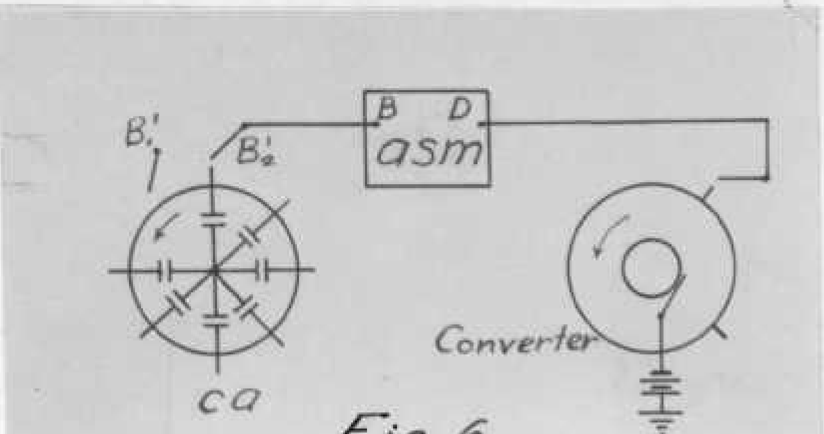


Fig. 6

The contact studs on the converter are maintained at a negative potential and the brushes connect to the addition terminal D of the asm. It is apparent that any other number might be placed on the abacus by merely arranging contact studs properly on the converter. For converting from base ten to base two, the most convenient numbers to represent on the converter are 1, 2, 3...10, 20, 30...100, 200, 300..., up to 10^{15} in the present case; that is, the units, tens, hundreds, etc. This requires 136 combinations of contacts, which in the present machine are mounted radially in the surface of a cylinder ten inches long. To place any number on an abacus, it is only necessary to close a switch (by means of the base ten card or keyboard) connecting the proper converter brush to the asm.

Base ten card reading mechanism. In order to allow large amounts of information to be rapidly placed in the machine it was decided to use the standard cards of the punched-card tabulating equipment. These cards contain sets of hole-possibilities arranged in rows and columns, the position in a column indicating the digit and the position in a row indicating the place in the number. Thus the number 1582, for instance, is represented by holes punched as shown in Figure 7.

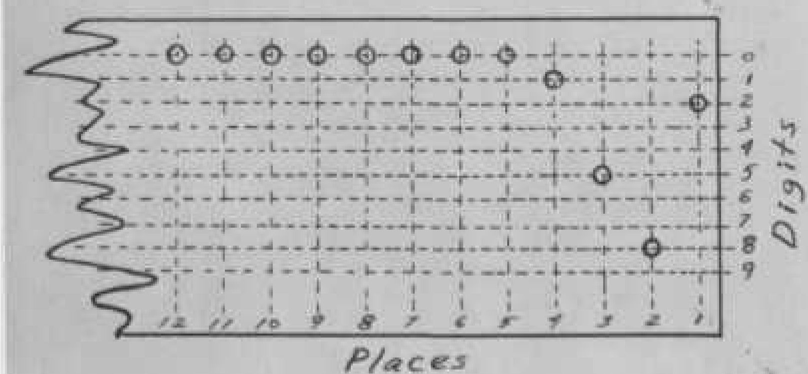


Fig. 7

In reading such a card into the present machine the card is placed on a stationary matrix consisting of an insulating plate which contains rows and columns of contacts, the contacts being spaced to correspond to the hole-possibilities on the card. Each contact is connected to the corresponding converter brush; i.e., the contact in the units row and the number one column connects to the number one brush, etc. A movable set of electrically-connected brushes is arranged to slide along over the card in such a manner that wherever there is a hole in the card, the brush in that column touches the contact below the hole. The movement of the brush assembly takes place during the vacant part of the cycle; during the active part of the cycle the assembly remains stationary over one of the rows, with one of its brushes touching the uncovered contact in that row.

Suppose the number 1582, used in the previous illustration, is to be transferred to an abacus. The sequence of events is as follows: During the first cycle after the card has been placed on the matrix, the brush assembly is above the units row, and the number two brush is in contact, closing the circuit between the asm addition terminal and the number two converter brush. At the end of this cycle the abacus has received charges from the converter corresponding to the number two. The brush assembly moves to the tens row during the vacant period, and for the next cycle the eight brush touches the contact which is connected to the 80 convertor. Owing to the action of the asm, 80 is added to the two already in the abacus. During the next cycle 500 is added, and during the next, 1000, so that after four cycles the abacus carries the desired number, 1582. Use of 80-place cards and a five-section matrix allows five 15-place numbers to be entered simultaneously and the entire process requires only 15 seconds.

A base-ten keyboard, which permits manual operation, functions in a manner similar to the card system but this is of relatively little importance, and its details will not be included here.

Base-ten dials. The base-ten dials for recording final results consist of 15 dials each containing numerals from 0 through 9 and each corresponding to one place in the number. These dials are electromagnetically driven and interlocked in a manner to be explained later.

In transferring numbers from the abacus to the dials a division process by powers of ten is carried out, making use of the fact that any base-ten number of N places may be expressed as $a.10^N + b.10^{N-1} + c.10^{N-2} + \ldots$, where a, b, c are the digits of the number. Arrangements are made to automatically subtract from the number on the abacus the base two equivalents of powers of ten (through the conversion table) starting with 10^N. The N dial records the number of times (one time per cycle) that 10^N is subtracted. When 10^N has been subtracted once too many times, the number remaining on the abacus becomes negative and, acting through a special control circuit, causes the asm to cease subtracting 10^N and to begin adding 10^{N-1}. (This is preferable to the more obvious and usual step of adding 10^N back on to the number once and then subtracting 10^{N-1}). The (N-1) dial records nine minus the number of times 10^{N-1} is added, and by a mechanical interlock drops the N dial back one step to its correct reading. When 10^{N-1} has been added enough times to make the number on the abacus positive, the asm is shifted to subtract and begins subtracting 10^{N-2}. The same process is repeated until the number on the abacus has disappeared, at which time the base-ten dials will show the correct base-ten expression for the number formerly on abacus as a base-two number.

6. Miscellaneous Details

It must be remembered that the foregoing description has necessarily omitted a considerable number of small details, which will be listed here.

High-speed (900 rpm) commutators are used in conjunction with the carry-over devices to time the charging and reading, for the "boosting" of the abaci and carry-over condensers during charging periods, and for periodically discharging the condensers on the addition input of the asm's. The low-speed (60 rpm) commutators discharge the carry-over devices at the end of each cycle to prevent carry-over into the first place from the last, and provide impulses for holding switches, etc.

The automatic controls shift numbers around the km, change the asm's from add to subtract as required, indicate whether any given abacus carries a positive or negative number, and halt operations whenever a result or sub-result is reached. In designing these controls free use has been made of vacuum tubes to obtain the high operating speeds which this machine requires.

The manual controls include power switches, a keyboard, switches for controlling the start of various operations (such as card punching or reading), routing switches to select the particular abacus on which any number is to be placed, and a flexible arrangement of plugs and jacks to provide for special setups.

7. Application of the Machine to Systems of Linear Algebraic Equations.

A few details will now be given of the application of this machine to the principal problem for which it is intended. The data giving the coefficients will come to the machine punched on standard base-ten cards. If there are thirty coefficients including the constant term six standard cards will be required for each equation. These numbers will be automatically converted to the base-two system and then transferred to the ka cylinder. The ca cylinder will then receive the coefficients of another equation. The negative coefficients on the base-ten cards will carry a special punching that will make the corresponding asm subtract them in the conversion process, thus yielding their complements. We thus have the coefficients of one equation on the ka's and another on the ca's. The switch S_2 is closed and a subtraction process is started and carried out until the control attached to one of the ca's indicates a carry through. Automatically the shift operates and addition takes place until there is again a carry through on the same ca, etc. This process is quite similar to the conversion from base two to base ten and in part the same controls are used. It serves to eliminate one coefficient from the record on the ca's and the results are punched on a base-two card. After the same coefficient has been eliminated from a sufficient number of pairs of equations the process is repeated with the control attached to another ca, thus eliminating another coefficient.

In the end there is left the coefficient of one unknown and the constant term. An elimination between this equation and one from the previous set that contains two unknowns yields an equation with the coefficient of another unknown and another constant term, etc. The quotient of the constant term by the unknown yields the value of the unknown in each case.

8. <u>Progress in Construction and Design.</u>

Substantial progress has been made in the construction of the machine, part of which may be seen in the accompanying photographs. Fig. 11 shows a general view of the entire machine as of August 8, 1940. This shows the welded angle-iron frame, the two abaci cylinders in place on their shaft, the rack into which all 30 asm's may be plugged, a voltage-regulated bias supply, the 100-ampere filament transformer and the d-c driving motor (to be replaced by a synchronous motor and a gear drive). Fig. 10 is a close-up view from the rear, showing clearly how the connections from the asm plug-in sockets are brough up through the channels in the rack to terminal blocks.

Top and bottom views of an asm are shown in Fig. 9. These units are extremely compact, measuring five inches by seven inches, yet containing seven tubes and 48 resistors. The 30 asm's and the 30 holder-shifter-circuits required for the entire machine have been completed (August 14, 1940).

A close-up view of one of the abaci cylinders is shown in Fig. 9. Each of the cylinders has been filled with 1600 small paper condensers, with the outer end of each condenser connected to a contact stud and the inner ends connected together and brought out through the mounting plates. The space near the periphery, in which the condensers are mounted, contains a high grade of wax for moisture protection. The inner space, about five inches in diameter, is empty. The manufacturer of the condensers has stated that since the condensers are being operated at about 10% of their rated voltage their life should be indefinitely long; however, it will be possible to replace any condenser in the remote event that a defect develops. As an additional precaution, two spare abaci have been placed in each cylinder.

A test setup of an abacus, asm, and converter was made in January, 1940. This arrangement performed perfectly and allowed actual tests under working conditions to be given the various components.

While a great deal of work is yet to be done the details of all that remains have been carefully thought out and in most cases actual working designs have been made. It is believed that the two major difficulties have been surmounted -- the perfection of the asm design and the electrical system of recording data.

Fig. 8

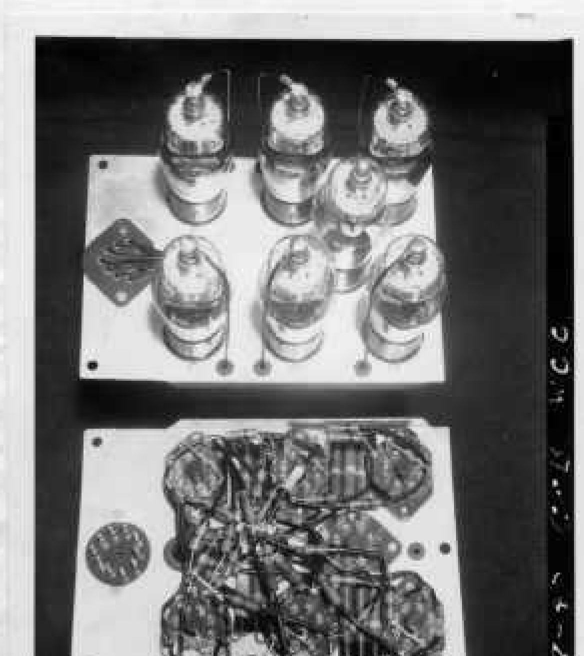


Fig. 9

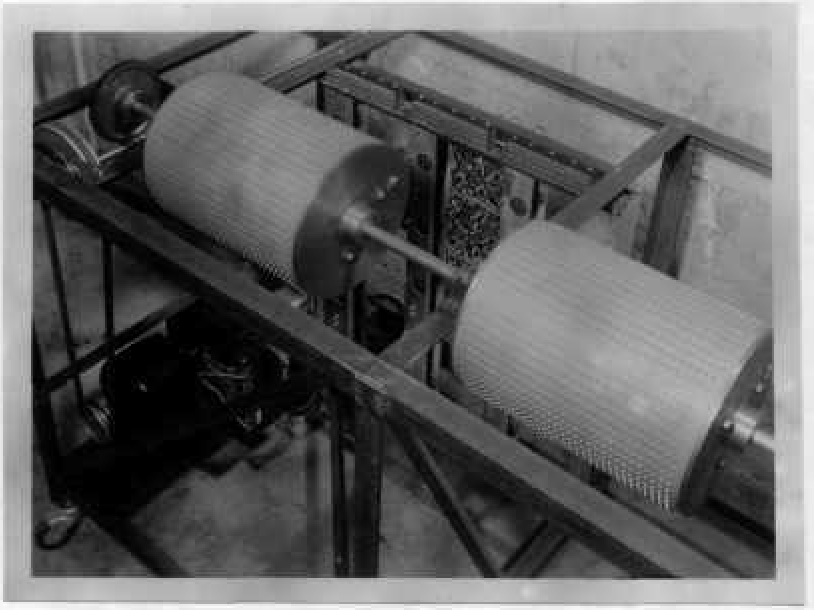


Fig. 10

Fig. 11

FINANCIAL STATEMENT

In the Spring of 1939 when it was desired to begin more active work on this project a sum of $650 was requested of and granted by the Iowa State College Research Council. During the winter and spring of 1940, $110 was allowed by the Dean of the Science Division of Iowa State College for the purchase of additional material. In the Spring of 1940 the Research Council made a further grant of $700 to continue work on the machine. On September 15, 1940 the account will roughly stand as follows:

Receipts

Research Council Spring, 1939	$650
Dean Gaskill Winter, 1940	110
Research Council Spring, 1940	700
	$1,460

Disbursements

Mr. Berry, Research Assistantship held 1939-40	450
Mr. Berry, full time services, Summer 1940	300
College Instrument Shop, services	90
Materials	290
Balance, Treas. Iowa State College	330
	$1,460

In addition Iowa State College has housed the project and furnished its various facilities free except for the services of the College Instrument Shop which is on a strict budgetary basis. It is expected that such aids will be continued. It will be noted that no part of the writer's personal services have been charged to this project.

It is the purpose of the writer to secure a grant of $5,000 in addition to the balance already alloted to the project, to be used during the next two years to complete the construction of this machine, and to test and perfect it under actual operating conditions. A tentative budget follows:

For personnel actively employed in the design, preparation of drawings, actual construction and testing of the machine.	$1900
For the services of the College Instrument Shop, or other similar organizations, in the execution of certain details of fine mechanical construction, for example the card handling mechanism and the card reading matrix.	1000
For the purchase of materials such as vacuum tubes, thyratrons, wire, condensers, resistors, brass bakelite and steel fabricated in standard forms, one synchronous motor, gearing, base-ten counter dials, keyboard, etc.	920
To establish two research assistantships of $540 each, the recipients to be graduate students in applied mathematics or mathematical physics, to apply the machine to actual problems and thus provide the background for the perfection of the device.	1080
For mechanical revisions necessitated by practical considerations, upkeep, repairs	430
	$5,330

An effort has been made to make this budget as accurate an estimate as possible of the cost of placing this machine into useful, productive operation. To refute the possible feeling that there has been too much provision for the revision and perfection of the machine after it is completed, one has only to inform himself of the number of rather excellent projects that have been orphaned because of insufficient funds for this purpose. At the same time, it is felt that the proposed budget is adequate. It has been checked by Mr. Clifford Berry as well as the writer and both are engineering graduates without illusions as to the cost of mechanical construction. However, it would have seemed absolutely impossible to the writer two years ago to have designed and constructed a computing machine of so large a capacity on so small a budget.