Proposed Automatic Calculating Machine

Howard Aiken

Aiken's formal proposal exists in at least three copies, all of which are in the Harvard University Archives. One is among the university's presidential papers for 1938, in a file marked "Physics." The last page is dated "Jan. 17, 1938" and signed "Howard Aiken." This document was formally transmitted to President J. B. Conant on 7 February 1938 by Professor Harry Mimno, whose covering letter to Conant mentions that Aiken had already made contact with IBM and that the two IBM engineers who had studied the proposal had found "the fundamental design" to be "practical." Clearly, the proposal had been written and submitted to IBM before 7 February 1938. On the first page of this copy of the proposal, someone has noted: "This was written in 1938 before construction was started." A second copy, in the files of the School of Engineering, is signed "Howard H. Aiken" in ink and is dated, in Aiken's hand, "January 17, 1938"—the same date that is on the president's copy, and apparently the day on which Aiken officially presented this document to the dean. The third copy, in the Aiken files, is dated in pencil, in a hand that has not been identified, "November 1937." The text is identical in all three copies, which were reproduced by some process. (They are not carbon copies of a typewritten document.)

There is evidence that the date of composition of Aiken's proposal (as opposed to the date of formal transmission to the president and the dean) is 1937. Reference 8 at the end of chapter 2 of the Manual of Operation for Mark I reads "H. H. Aiken, Proposed Automatic Calculating Machine (1937), p. 18, (privately distributed)." Since there are several references to IBM technology, it would appear that this proposal was prepared for Aiken's first contact with IBM: his meeting with James Wares Bryce, IBM's chief engineer, which took place in early November 1937. It does not seem likely that Aiken would have referred to IBM and its machines if this proposal had been written for his unfruitful meeting with George Chase of the Monroe Calculating Machines Company on 22 April 1937. (For additional evidence to support the date 1937, probably November, see Portrait.*)*

This landmark text, published in IEEE Spectrum *in 1964, has been reprinted from that journal in the three editions of Brian Randell's source book on the antecedents and early history of the computer,* The Origins of Digital Computers—Selected Papers *(third edition: Springer-Verlag, 1975). It has also been reprinted in at least one other anthology of writings concerning the computer.*

The original text, reprinted here, differs in some features from the version printed in IEEE Spectrum *and reprinted in Randell's anthology, which contains a number of* *alterations. For example, some of Aiken's mentions of "International Business Machines" were changed to "IBM machines." "International Business Machines Company," however, was kept as Aiken wrote it, as was Aiken's "MacLauren" for the name of the Scottish mathematician Colin Maclaurin. Also altered were the paragraphing and numbering of some sections. One section in which many alterations were made is titled "Present Conception of the Apparatus." In the edited version, the numbering and paragraphing were altered, and the displayed and numbered lists lost their numbers and were converted into paragraphs. The text printed below reproduces Aiken's original document word for word.*

I. Historical Instodution

The desire to economize time and mental effort in arithmetical computations, and to eliminate human liability to error, is probably as old as the science of arithmetic itself. This desire has led to the design and construction of a variety of aids to calculation, beginning with groups of small objects, such as pebbles, first used loosely, later as counters on ruled boards, and later still as beads mounted on wires fixed in a frame, as in the abacus. This instrument was probably invented by the Semitic races and later adopted in India, whence it spread westward throughout Europe and eastward to China and Japan.

After the development of the abacus no further advances were made until John Napier devised his numbering rods, or Napier's Bones, in 1617. Various forms of the Bones appeared, some approaching the beginning of mechanical computation, but it was not until 1642 that Blaise Pascal gave us the first mechanical claculating machine in the sense that the term is used today. The application of his machine was restricted to addition and subtraction, but in 1666 Samuel Moreland adapted it to multiplication by repeated additions.

The next advance was made by Leibnitz who conceived a multiplying machine in 1671 and finished its construction in 1694. In the process of designing this machine Leibnitz invented two important devices which still occur as components of modern calculating machines today; the stepped reckoner, and the pin wheel.

Meanwhile, following the invention of logarithms by Napier, the slide rule was being developed by Oughtred, John Brown, Coggeshall, Everard, and others. Owing to its low cost and ease of construction, the slide rule received wide recognition from scientific men as early as 1700. Further development has continued up to the present time, with ever increasing application to the solution of scientific problems requiring an accuracy of not more than three or four significant figures, and when the total bulk of the computation is not too great.

Particularly in engineering design has the slide rule proved to be an invaluable instrument.

Though the slide rule was widely accepted, at no time, however, did it act as a deterrent to the development of the more precise methods of mechanical computation. Thus we find the names of some of the greatest mathematicians and physicists of all time associated with the development of calculating machinery. Naturally enough, these men considered mechanical calculation largely from their own point of view, in an effort to devise means of scientific advancement. A notable exception was Pascal who invented his calculating machine for the purpose of assisting his father in computations with sums of money. Despite this widespread scientific interest, the development of modern calculating machinery proceeded slowly until the growth of commercial enterprises and the increasing complexity of accounting made mechanical computation an economic necessity. Thus the ideas of the physicists and mathematicians, who foresaw the possibilities and gave the fundamentals, have been turned to excellent purposes, but differing greatly from those for which they were originally intended.

Few calculating machines have been designed strictly for application to scientific investigations, the notable exceptions being those of Charles Babbage and others who followed him. In 1812 Babbage conceived the idea of a calculating machine of a higher type than those previously constructed, to be used for calculating and printing tables of mathematical functions. This machine worked by the method of differences, and was known as difference engine. Babbage's first model was made in 1822, and in 1823 the construction of the machine was begun with the aid of a grant from the British Government. The construction was continued until 1833 when state aid was withdrawn after an expenditure of nearly £ 20,000. At the present time the machine is in the collection of the Science Museum, South Kensington.

In 1934 George Scheutz of Stockholm read the description of Babbage's difference engine and started the construction of a similar machine with the aid of a governmental grant. This machine was completed and utilized for printing mathematical tables. Then followed several other difference engines constructed and designed by Martin Wiberg in Sweden, G. B. Grant in the United States, Leon Bolleé in France, and Percy Ludgate in Ireland. The last two, however, were never constructed.

After abandoning the difference engine, Babbage devoted his energy to the design and construction of an analytical engine of far higher powers than the difference engine. This machine, intended to evaluate any algebraic formulae by the method of differences, was never completed, being too ambitious for the time. It pointed the way, however, to the modern punched card type of calculating machine since it was intended to use for its control perforated cards similar to those used in the Jacquard loom.

Since the time of Babbage the development of calculating machinery has continued at an increasing rate. Key driven calculators designed for single arithmetical operations such as addition, subtraction, multiplication, and division, have been brought to a high degree of perfection. In large commercial enterprizes, however, the volume of accounting work is so great that these machines are no longer adequate in scope.

Hollerith, therefore, returned to the punched card formerly used by Babbage and with it laid the ground work for the development of tabulating, counting, sorting, and arithmetical machinery such as is now widely utilized in industry. The development of electrical apparatus and technique found application in these machines as manufactured by the International Business Machines Company, until today many of the things Babbage wished to accomplish are being done daily in the accounting offices of industrial enterprizes all over the world.

As previously stated, these machines are all designed with a view to special applications to accounting. In every case they are concerned with the four fundamental operations of arithmetic, and not with operations of algebraic character. Their existence, however, makes possible the construction of an automatic calculating machine specially designed for the purposes of the mathematical sciences.

II. The Need for More Powerful Calculating Methods in the Mathematical and Physical Sciences

It has already been indicated that the need for mechanical assistance in computation has been felt from the beginning of science, but at the present time this need is greater than ever before. The intensive development of the mathematical and physical sciences in recent years has included the definition of many new and useful functions nearly all of which are defined by infinite series or other infinite processes. Most of these are inadequately tabulated and their application to scientific problems is thereby retarded.

The increased accuracy of physical measurement has made necessary more accurate computation in physical theory, and experience has

shown that small differences between computed theoretical and experimental results may lead to the discovery of a new physical effect, sometimes of the greatest scientific and industrial importance.

Many of the most recent scientific developments, including such devices as the thermionic vacuum tube, are based on nonlinear effects. Only too often the differential equations designed to represent these physical effects correspond to no previously studied forms, and thus defy all methods available for their integration. The only methods of solution available in such cases are expansions in infinite series and numerical integration by iterative methods. Both these methods involve enormous amounts of computational labor.

The present development of theoretical physics through Wave Mechanics is based entirely on mathematical concepts and clearly indicates that the future of the physical sciences rests in mathematical reasoning directed by experiment. At the present time there exist problems beyond our ability to solve, not because of theoretical difficulties, but because of insufficient means of mechanical computation.

In some fields of investigation in the physical sciences, as for instance in the study of the ionosphere, the mathematical expressions required to represent the phenomena are too long and complicated to write in several lines across a printed page, yet the numerical investigation of such expressions is an absolute necessity to our study of the physics of the upper atmosphere, and on this type of research rests the future of radio communication and television.

The roots of transcendental equations and algebraic equations above the second degree can be obtained only by successive approximations, and if an accuracy of ten significant figures is required the numerical labor in many cases may be all but prohibitive.

These are but a few examples of the computational difficulties with which the physical and mathematical sciences are faced, and to these may be added many others taken from astronomy, the theory of relativity, and even the rapidly growing science of mathematical economy. All these computational difficulties can be removed by the design of suitable automatic calculating machinery.

III. Points of Difference between Punched Card Accounting Machinery and Calculating Machinery as Required in the Sciences

The features to be incorporated in calculating machinery specially designed for rapid work on scientific problems, and not to be found in calculating machines as manufactured for accounting purposes, are the following.

1. Ordinary accounting machines are concerned entirely with arithmetical problems, while machines designed for mathematical purposes must be able to handle both positive and negative quantities.

2. For mathematical purposes, calculating machinery should be able to supply and utilize a wide variety of transcendental functions, as the trigonometric functions; elliptic, Bessel, and probability functions; and many others. Fortunately not all these functions occur in a single computation; therefore a means of changing from one function to another may be designed and the proper flexibility provided.

3. Most of the computations of mathematics, as the calculation of a function by series, the evaluation of a formula, the solution of a differential equation by numerical integration, etc., consist of repetitive processes. Once a process is established it may continue indefinitely until the range of the independent variables is covered, and usually the range of the independent variables may be covered by successive equal steps. For this reason calculating machinery designed for application to the mathematical sciences should be fully automatic in its operation once a process is established.

4. Existing calculating machinery is capable of calculating $\phi(x)$ as a function of x by steps. Thus, if x is defined in the interval $a<x<b$ and $\phi(x)$ is obtained from x by a series of arithmetical operations, the existing procedure is to compute step (1) for all values of x in the interval $a<x<b$. Then step (2) is accomplished for all values of the result of step (1), and so on until $\phi(x)$ is reached. This process, however, is the reverse of that required in many mathematical operations. Calculating machinery designed for application to the mathematical sciences should be capable of computing lines instead of columns, for very often, as in the numerical solution of a differential equation, the computation of the second value in the computed table of a function depends on the preceding value or values.

Fundamentally, these four features are all that are required to convert existing punched card calculating machines such as those manufactured by the International Business Machine Company into machines specially adapted to scientific purposes. Because of the greater complexity of scientific problems as compared to accounting problems, the number of arithmetical elements involved would have to be greatly increased.

IV. The Mathematical Operations which should be Included

The mathematical operations which should be included in an automatic calculating machine are:

1. The fundamental operations of arithmetic

a. addition

b. subtraction

c. multiplication

d. division

2. Positive and negative numbers

3. Parentheses and brackets

a. ( ) + ( )

b. [( ) + ( )] · [( ) + ( )]

c. Etc.

4. Powers of numbers

a. Integral

b. Fractional

5. Logarithms

a. Base 10

b. All other bases by multiplication

6. Antilogarithms or exponential functions

a. Base 10

b. Other bases

7. Trigonometric functions

8. Anti-trigonometric functions

9. Hyperbolic functions

10. Anti-hyperbolic functions

11. Superior transcendentals

a. Probability integral

b. Elliptic function

c. Bessel function

With the aid of these functions the processes to be carried out should be:

12. Evaluation of formulae and tabulation of results

13. Computation of series

a. Finite

b. Infinite

14. Determination of the real roots of equations

15. Solution of ordinary differential equations of the first and second order

16. Numerical integration of empirical data

17. Numerical differentiation of empirical data

V. The Mathematical Means of Accomplishing the Operations.

The purpose of this section is to describe the mathematical processes which may be made the basis of design of an automatic calculating machine. In the case of every operation considered it should be noted that the formulae suggested reduce the operation to a repetitive sequence.

1. The fundamental arithmetical operations require no comment, as they are already available, save that all the other operations must eventually be reduced to these in order that a mechanical device may be utilized.

2. Fortunately the algebra of positive and negative signs is extremely simple. In any case only two possibilities are offered. Later on it will be shown that these signs may be treated as numbers for the purposes of mechanical calculation.

The use of parentheses and brackets in writing a formula requires that the computation must proceed piecewise. Thus, a portion of the result is obtained and must be held pending the determination of some other portion, and so on. This means that a calculating machine must be equipped with means of temporarily storing numbers until they are required for further use. Such means are available in counters.

4. Integral powers of numbers may be obtained by successive multiplication, and fractional powers by the method of iteration. Thus, if it is required to find $5^{1/3}$,

$$y = f(x) = x^3 - 5 \tag{a}$$

and

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} \tag{b}$$

$$x_n = x_{n-1} - \frac{x_{n-1}^3 - 5}{3x_{n-1}^2} \tag{c}$$

or

$$x_n = \frac{2}{3}x_{n-1} + \frac{5}{3x_{n-1}^2} \qquad \text{(d)}$$

Let

$$x_0 = 2$$

$$x_1 = \frac{4}{3} + \frac{5}{12} = \frac{21}{12} \qquad \text{(e)}$$

$$x_2 = \frac{42}{36} + \frac{5 \times 144}{3 \times 441} = 1.166 \ + 0.544$$

$$= 1.710$$

which is the cube root of 5 to four significant figures. In general the rth root of θ is given by the iteration of the expression:

$$x_n = \left(1 - \frac{1}{r}\right)x_{n-1} + \frac{\theta}{r x_{n-1}^{r-1}} \qquad \text{(f)}$$

Finally, if r is not an integer recourse may be had to the mechanical table of logarithms later to be described.

5. To supply a mechanical device with a complete mathematical function over a wide range of values would require an impossible amount of apparatus. To avoid this difficulty several artifices may be employed. In the case of logarithms, let it be required to find

$$y = \log_{10} x \qquad \text{(a)}$$

Then

$$x = 10^y$$

$$= 10^{a + \frac{b}{10} + \frac{c}{100} + \frac{d}{1000} + \cdots} \qquad \text{(b)}$$

where a, b, c, . . . are all integers. If x is restricted to have values no larger than 10^{10}, then

$$0 \leq a, b, c, \ldots \leq 9 \qquad \text{(c)}$$

Equation (b) may then be written

$$x = 10^a \times (10^{\frac{1}{10}})^b \times (10^{\frac{1}{100}})^c \times (10^{\frac{1}{1000}})^d \times \cdots \qquad \text{(d)}$$

We may now form a table consisting of 100 numbers:

	0	1	2	3	4	5
10	1	10.000	100.00	1000.0	...	
$10^{1/10}$	1	1.2589	1.585	1.995	...	
$10^{1/100}$	1	1.0238	1.0471	1.0715	...	
$10^{1/1000}$	1	1.0023	1.0046	1.006	...	
$10^{1/10000}$	...	...				

giving the integral powers from 0 to 9 inclusive of 10, $10^{1/10}$, $10^{1/100}$, etc. Then, if it is required to find $\log_{10}2104$, for instance, choose the largest number in the first row which, when divided into 2104, still leaves a result greater than unity. Thus,

$$\frac{2.104}{1000} = 2.104 - - - 3$$

where 1000 was taken from the 3rd column. Continuing,

$$\frac{2.104}{1.995} = 1.054 - - - 3$$

$$\frac{1.054}{1.0471} = 1.006 - - - 2$$

$$\frac{1.006}{1.006} = 1.000 - - - 3$$

Hence,

$$\log_{10}2104 = 3.323; \qquad \text{(e)}$$

this is correct to the last figure.

Thus it is seen that the computation of 10 significant figure logarithms may be reduced to ten discriminations, each in a field of ten, and eight divisions; eight because the first consists of moving the decimal point, a process as effortless in mechanical as in mental computation, and the last division need not be carried out.

6. The process of finding anti-logarithms may be reduced to a reversal of the logarithmic process. Thus, if

$$y = 10^x \tag{a}$$

then

$$y = 10^{a+\frac{b}{10}+\frac{c}{100}+..} \tag{b}$$

$$= (10)^a \times (10^{\frac{1}{10}})^b \times (10^{\frac{1}{100}})^c \ldots \tag{c}$$

and repetitive discrimination and multiplication suffices.

7. The trigonometric functions most commonly used are the sine and cosine, and from these all other trigonometric functions may be computed easily. Either of these functions may be computed from the other, but in the expansion of Fourier series both sines and cosines are required. Therefore, it seems worth while to consider mechanical means of computing both the functions.

On expanding $\sin(a + h)$ by MacLauren's Theorem,

$$\sin(a{+}h) = \sin a + \frac{\cos a}{1}h - \frac{\sin a}{2}h^2 - \frac{\cos a}{6}h^3 + \frac{\sin a}{24}h^4 - \ldots \tag{a}$$

If now,

$$\theta = a + h \tag{b}$$

and

$$-\pi/2 \leq \theta \leq \pi/2 \tag{c}$$

twenty values of a may be chosen, as

$$a = \pi/2,\ 9\pi/20,\ 4\pi/5,\ \ldots.\ -9\pi/20 \tag{d}$$

Then the maximum value of h is

$$h = \pi/20 = 0.15729... \tag{e}$$

and ten terms of the series suffice for determining $\sin\theta$ significant figures, at most. On the average approximately five terms are sufficient. The process of computing sines is thus reduced to discriminations of one number in a field of twenty, and the computation of a series of at most ten terms.

The process for computing the cosine is exactly the same, and from these all other trigonometric functions may be determined arithmetically by

$$\csc\theta = 1/\sin\theta \tag{f}$$

$$\sec\theta = 1/\cos\theta \tag{g}$$

$$\tan\theta = \sin\theta/\cos\theta \tag{h}$$

Thus a field of 200 numbers of sufficient to supply all trigonometric functions.

8. The inverse trigonometric functions may also be determined by MacLauren's Theorem, but since $\sin^{-1}\theta$ and $\tan^{-1}\theta$ occur more often than any other inverse trigonometric function, these should be selected and any others computed from them.

9. Similar methods might be applied to the computation of the hyperbolic functions, but it is questionable if special apparatus should be initially installed for their determination since the hyperbolic functions may all be defined in terms of exponentials computable from the logarithmic device already suggested.

10. Similar comments apply to the inverse hyperbolic functions.

11. A great many functions may be similarly treated, and if the design of the automatic calculating machine proceeds so that a given device can be changed from one function to another rapidly, all such functions may be included in the scope of the machine. Means of accomplishing this will be suggested later.

12. Given a suitable supply of transcendental functions, the evaluation of formulae is reduced to arithmetic. If a formula is to be evaluated for a wide range of the independent variable, the process becomes repetitive. Means for accomplishing this will be discussed later.

13. The computation of closed series such as

$$y = a_0 + a_1x + a_2\,x^2 + a_3\,x^3 \tag{a}$$

is most easily accomplished by the sequence:

a_3

a_3x

$a_3x + a_2$

$$a_3x^2 + a_2x \tag{b}$$

$a_3x^2 + a_2x + a_1$

$a_3x^3 + a_2x^2 + a_1x$

$a_3x^3 + a_2x^2 + a_1x + a_0 = y$

In the case of infinite series the computation may be reduced to successive multiplications and additions. Thus, if

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ldots \qquad \text{(c)}$$

$$= a_0$$

$$+ \frac{a_1}{a_0} x \cdot a_0$$

$$+ \frac{a_2}{a_1} x \cdot a_0$$

$$+ \frac{a_3}{a_2} x \cdot \frac{a_2}{a_1} x \cdot \frac{a_1}{a_0} x \cdot a_0$$

$$+ \frac{a_4}{a_3} x \cdot \frac{a_3}{a_2} x \cdot \frac{a_2}{a_1} x \cdot \frac{a_1}{a_0} x \cdot a_0 \qquad \text{(d)}$$

$$+ \ldots$$

and

$$y = A_0$$

$$+ A_1 x \cdot A_0$$

$$+ A_2 x \cdot A_1 x \cdot A_0$$

$$+ A_3 x \cdot A_2 x \cdot A_0$$

$$+ A_4 x \cdot A_3 x \cdot A_2 x \cdot A_1 x \cdot A_0 \qquad \text{(e)}$$

where

$$A_0 = a_0;\ A_1 = a_1/a_0;\ A_2 = a_2/a_1;\ \ldots \qquad \text{(f)}$$

Thus each term of the series is obtained from the last through multiplication by a coefficient and the value of the independent variable.

14. Any mechanical device that can evaluate formulae can also determine the real roots of algebraic and transcendental equations provided only that in the evaluation of the formulae the successive values of the independent variable are the successive values of the dependent variable computed; thus, consider

$$x + \log_{10} x = 1/2 \qquad \text{(a)}$$

given that x is in the neighborhood of 1/2. A succession of eleven approximations suffices to give

$$x = 0.672384 \qquad \text{(b)}$$

Or, let the equation be the famous cubic of Wallis,

$$x^3 - 2x - 5 = 0 \qquad \text{(c)}$$

the iterative equation is

$$x_n = x_{n-1} - \frac{x_{n-1}^3 - 2x_{n-1} - 5}{3x_{n-1}^2 - 2} \qquad \text{(d)}$$

Three approximations give

$$x = 2.09455148\ldots \qquad \text{(e)}$$

The root of this equation has been computed to 150 significant figures. Note that again the process is purely repetitive after being started.

15. The solution of ordinary differential equations of any order can usually be accomplished to any degree of accuracy by expansion into infinite series by MacLauren's Theorem for any specified boundary demands. Under certain circumstances the series may be rapidly convergent and the method offers excellent means for numerical solution.

However, when the equation has complicated functions of x as coefficients of the various derivatives of y, and the independent variable itself occurs in complicated functions, the various successive derivatives necessary to the series expansion may involve a prohibitive amount of labor. For such cases various methods of numerical solution have been devised, such as those of Adams, Runge-Kutta, and others.

Of these, the method of Runge-Kutta is probably best adapted to mechanical computation because the method of solution depends entirely on the evaluation of a repetitive sequence. Thus, if

$$\frac{dy}{dx} = f(x,y) \qquad \text{(a)}$$

and

$$K_1 = f(x_0, y_0)\Delta x$$

$$K_2 = f\left(x_0 + \frac{\Delta x}{2}, y_0 + \frac{K_1}{2}\right)\Delta x \qquad \text{(b)}$$

$$K_3 = f\left(x_0 + \frac{\Delta x}{2}, y_0 + \frac{K_2}{2}\right)\Delta x$$

$$K_4 = f(x_0 + \Delta x, y_0 + K_3)\,\Delta x$$

Then,

$$\Delta y = (x_0 + \Delta x, y_0 + K_3)\,\Delta x$$

and

$$y_1 = y_0 + \Delta y$$

$$x_1 = x_0 + \Delta x \qquad \text{(c)}$$

The process may now be repeated to find x_2, y_2, and so on. The inherent error of this process is of the order of Δx^5; hence, if Δx is taken as 0.1, the solution will be correct to the fourth place of decimals, and doubtful in the fifth.

The method can be applied to simultaneous equations of the first order, and hence to second order equations.

Since the method involves nothing other than the evaluation of formulae, a mechanical device suitable for such evaluation is prepared to perform this type of numerical integration.

16. The numerical integration of empirical data may be carried out by the rules of Simpson, Weddle, Gauss, and others. All these rules involve sums of successive values of y multiplied by specified numerical coefficients. Hence the only new mechanical component involved in a means of mechanically introducing a list of numbers. Means of accomplishing this will be discussed later.

17. Numerical differentiation of empirical data is best accomplished by means of a difference formula. Most experimental observations are of such an accuracy that fifth differences may be neglected by taking observations sufficiently close together. If, then, all differences above the fifth may be neglected, the process of numerical differentiation may be carried out by a fifth difference engine such as originally designed by Babbage. Such a device can, however, be assembled from standard addition-subtraction machines with but a few changes. The

differentiating apparatus would also be applicable to many other problems. In fact, most of the problems already discussed may under certain circumstances be solved by application of difference formulae.

VI. Mechanical Considerations

In the last section it was shown that even complicated mathematical operations may be reduced to a repetitive process involving the fundmanetal rules of arithmetic. At the present time the calculating machine of the International Business Machines Company are capable of carrying out such operations as:

$$A + B = F$$

$$A - B = F$$

$$AB + C = F$$

$$AB + C + D = F \qquad \text{(a)}$$

$$A + B + C = F$$

$$A - B - C = F$$

$$A + B - C = F$$

In these equations, A, B, C, D are tabulations of numbers on punched cards, and F, the result, is also obtained through punched cards. The F cards may then be put through another machine and printed or utilized as A, B, . . . cards in another computation.

Changing a given machine from any of the operations (a) to any other is accomplished by means of electrical wiring on a plug board. In the hands of a skilled operator such changes can be made in a few minutes.

No further effort will be made here to describe the mechanism of the International Business Machines. Suffice it to say that all the operations described in the last section can be accomplished by these existing machines when equipped with suitable controls, and assembled in sufficient number. The whole problem of design of an automatic calculating machine suitable for mathematical operations is thus reduced to a problem of suitable control design, and even this problem has been solved for simple arithmetical operations.

The main features of the specialized controls are machine switching and replacement of the punched cards by continuous perforated tapes.

In order that the switching sequence can be changed quickly to any possible sequence the switching mechanism should itself utilize a paper tape control in which mathematical formulae may be represented by suitably disposed perforations.

VII. Present Conception of the Apparatus

At present the automatic calculator is visualized as a switchboard on which are mounted various pieces of calculating machine apparatus. Each panel of the switchboard is given over to definite mathematical operations.

1. International Business Machines utilize two electric potentials, 120 a.c. for motor operation, and 32 volts d.c. for relay operation, etc. A main power supply panel would have to be provided including control for a 110 volt, a.c./32 volt d.c. motor generator and adequate fuse protection for all circuits.

2. Master Control Panel: The purpose of this control is to route the flow of numbers through the machines and to start operation. The processes involved are

a. Deliver the number in position (x) to position (y)

b. Start the operation for which position (y) is intended.

The master control must itself be subject to interlocking to prevent the attempt to remove a number before its value is determined, or to begin a second operation in position (y) before a previous operation is finished.

It would be desirable to have four such master controls, each capable of controlling the entire machine or any of its parts. Thus, for complicated problems the entire resources could be thrown together; for simple problems, fewer resources are required and several problems could be in progress at the same time.

3. The progress of the independent variable in any calculation would go forward by equal steps subject to manual readjustment for change in the increment. The easiest way to obtain such an arithmetical sequence is to supply a first value, x_0, to an adding machine, together

with the increment Δx. Then successive additions of Δx will give the sequence desired.

There should be four such independent variable devices in order to

a. Calculate formulae involving four variables,

b. Operate four master controls independently.

4. Certain constants: many mathematical formulae involve certain constants such as ε, π, $\log_{10} \varepsilon$, and so forth. These constants should be permanently installed and available at all times.

5. Mathematical formulae nearly always involve constant quantities. In the computation of a formula as a function of an independent variable these constants are used over and over again. Hence the machine should be supplied with 24 adjustable number positions for these constants.

6. In the evaluation of infinite series the number 24 might be greatly exceeded. To take care of this case it should be possible to introduce specific values by means of a perforated tape, the successive values being supplied by moving the tape ahead one position. Two such devices should be supplied.

7. The introduction of empirical data for non-repetitive operations can be accomplished best by standard punched card magazine feed. One such device should be supplied.

8. At various stages of a computation involving parentheses and brackets it may be necessary to hold a part of the result pending the computation of some other part. If results are held in the calculating unit these elements are not available for carrying out succeeding steps. Therefore it is necessary that numbers may be removed from the calculating units and temporarily stored in storage positions. Twelve such positions should be available.

9. The fundamental operations of arithmetic may be carried on three machines.

a. Addition and subtraction

c. Multiplication

d. Division.

Four units of each should be supplied in addition to those directly associated with the transcendental functions.

The permanently installed mathematical functions should include

10. Logarithms.

11. Anti-logarithms

12. Sines

13. Cosines.

14. Inverse Sines.

15. Inverse tangents.

16. Two units for MacLauren Series expansion of other functions as needed.

17. In order to carry out the process of differentiation and integration on empirical data, adding and subtracting accumulators should be provided sufficient to compute out to fifth differences.

18. All results should be printed, punched in paper tapes, or in cards at will. Final results would be printed. Intermediate results would be punched in preparation for further calculations.

The above is a rough outline of the apparatus required, and it is believed that this apparatus, controlled by automatic switching, would care for most of the problems encountered.

VIII. Probable Speed of Computation

An idea of the speed attained by the International Business Machines can be had from the following tabulation of multiplication in which 2×8 refers to the multiplication of an 8 significant figure number by a 2 significant figure number, zeros not counted.

	Products per hour
2×8	1500
3×8	1285
4×8	1125
5×8	1000
6×8	900
7×8	818
8×8	750

In the computation of 10 place logarithms the average speed would be about 90 per hour. If all the 10 place logarithms of the natural numbers from 1000 to 100,000 were required, the time of computation would be approximately 1100 hours, or 50 days, allowing no time for addition or printing. This is justified since these operations are extremely rapid and can be carried out during the multiplying time.

IX. Suggested Accuracy

Ten significant figures has been used in the above examples. If all numbers were to be given to this accuracy it would be necessary to provide 23 number positions on most of the computing components, 10 to the left of the decimal point, 12 to the right, and one for plus and minus. Of the twelve to the right two would be guard places and thrown away.

X. Ease of Publication of Results

As already mentioned, all computed results would be printed in tabular form. By means of photo-lithography these results could be printed directly without type setting or proof reading. Not only does this indicate a great saving in the publishing of mathematical functions, but it also eliminates many possibilities of error.